GLACIAL SURVEYS IN WESTERN CANADA

BY

G. KONECNY

Reprinted from
PHOTOGRAMMETRIC ENGINEERING
January
1964

*Glacial Surveys in Western Canada**

GOTTFRIED KONECNY,
Assoc. Prof., Dept. of Civil Engineering,
Div. of Surveying Engineering,
Univ. of New Brunswick, Fredericton, N. B., Can.

ABSTRACT: *The correlation between climate and glacial changes has led to a world-wide observation program of glaciers typical for advance or retreat.*

In Western Canada, accurate glacier mapping was started by aerial photogrammetry during the last decade. A research project of the University of New Brunswick applied terrestrial photo-theodolite surveys to glacial mapping and to the determination of speed profiles in 1962.

The paper compares economy and accuracy of terrestrial versus aerial photogrammetric surveys of the Athabaska glacier area. Methods for determining glacial retreat are discussed along with the results obtained.

1. INTRODUCTION

THE study of glaciers is of interest to various scientific fields and has practical significance for the determination of water resources and the study of climate. Glacial surveys provide valuable information to the glaciologist. They have two main objectives: 1) the study of ice velocity of the moving glacier and 2) the determination of volume changes due to access of accumulation of snow or due to melting of ice.

The methods employed in glacial surveys basically do not differ from those in ordinary surveys but they require practically instantaneous procedures, since glaciers are moving objects. Because of this, glaciological surveys and photogrammetry since their early history have had a very intimate relationship. One of the pioneers in photogrammetry, Sebastian Finsterwalder of Munich, as early as 1890 developed analytical photogrammetric methods primarily for the purpose of glacier surveys.

Since about 1910, when phototheodolites and corresponding plotting instruments became available, terrestrial photogrammetry has been used exclusively for the mapping of glaciers in Europe and on glacial expeditions conducted by European glaciologists (Colcord 1957, pp. 552–557) (Finsterwalder 1931, pp. 252–263; 1952; 1953, pp. 189–239; 1957, pp. 520–524) (Haumann 1960, pp. 388–397).

Aerial Photography for glacial studies was introduced mainly in North America, and since then several glaciers of large extent have been mapped by aerial photogrammetry all around the globe (Blachut, pp. 484–492) (Case, 1958, pp. 815–821; 1960) (Finsterwalder 1958, pp. 11–12) (Haumann pp. 74–110; 1960, pp. 388–397; 1958, pp. 71–88). But stadia and intersection methods or trilateration and leveling methods also continue to be employed as well as terrestrial photogrammetry.

GOTTFRIED KONECNY

It is the purpose of this paper to discuss the various requirements for glacial surveys and the limitations of the different survey methods employed. This will be illustrated by

---

* Presented at March 24–30, 1963 ASP-ACSM Convention, Hotel Shoreham, Washington, D. C.

the example of a glacial survey project in Western Canada.

2. Methods for Determining Glacier Velocity

a) PLAQUE LINE SURVEYS

Plaque lines are used to observe glacier velocity. The plaques are placed in profiles across or along the glacier. Their position at the beginning and at the end of the time interval is determined by stadia traversing or by intersection from fixed points on the side of the glacier. Frequently the elevation is fixed also by stadia, trigonometric or ordinary leveling.

The method is modified in Arctic areas where quadrilateral chains are measured by tellurometer trilateration and by ordinary leveling.

The accuracy of the survey depends upon the motion of the glacier during the time of measurement. Generally this motion is negligible and the accuracy will correspond to the method employed.

Plaque line surveys make possible studying glacier motion over long time intervals, generally a year or longer. They have the disadvantage that some plaques often cannot be recovered because they have melted, or have been covered by snow. They also may change their position due to melting or pressures within the ice. Plaque line surveys are therefore not suitable for short time intervals.

b) TERRESTRIAL PHOTOGRAMMETRY

Terrestrial photogrammetry can be used more successfully and economically if suitable exposure stations overlooking the glacier perpendicular to its direction of flow can be found. The phototheodolite stations whenever possible, should be located parallel to the glacier in order to avoid unnecessary turning away of the direction of photography from the normal to the base. The base length is ideally chosen as one-fifth to one-tenth of the average distance to the glacier in order to avoid difficulties with stereovision, while still maintaining a high intersection accuracy.

If the phototheodolite is leveled and directionally oriented with respect to the base, exposures *A(1)* and *B(1)* at both stations *A* and *B* will permit plotting on a stereo plotter any identifiable point on the glacier or the ground co-ordinates of such a point can be calculated after its photo co-ordinates have been measured on a stereocomparator or by a simple stereometer:

The formulae for such calculations are derived in most European texts of photogrammetry (Finsterwalder 1952) (Zeller 1947). They are listed in Figures 1 and 2 with the sign definitions of the Wild-P30 Phototheodolite for normal, averted, convergent and inclined photography. Figure 2 also shows how a change in direction is introduced in the Wild A5 or A7 autograph. It further lists the accuracy of point co-ordinates determined by terrestrial photogrammetry.

Glacial Motion from terrestrial photographs can be determined by two methods (Finsterwalder 1931, pp. 252–263):

A. Stereoscopic Intersection Method[1]

If the point determined by photos *A(1)* and *B(1)*, taken from stations *A* and *B* at the time (1) can be re-identified on subsequent photos *A(2)* and *B(2)* taken from the same stations, its motion components Δx, Δy, and Δz during the time interval can be determined from the differences in their co-ordinates. This method has the advantage of determining all co-ordinate components. It also does not depend on the recoverability of the exposure stations.

B. Motion Parallax Method

However, direct motion parallax measurement from photos *A(1)* and *A(2)* from the same station *A* results in higher accuracy for the determination of the x and z components of motion. (See Figure 3.)

Since the photographs are taken from the same station with the same orientation the parallax dpx will directly represent the x component of motion. With the plates rotated by 90°, the z component can also be measured stereoscopically. The x, y, z co-ordinates of the point are determined as for method *A* using either photos *A*(1), *B*(1), or *A*(2), *B*(2).

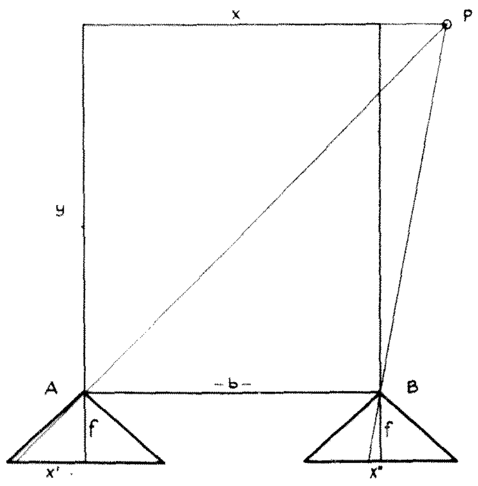
Method *B* is by $\sqrt{2}(m_x'/m_{px})$ more accurate than Method *A*. m_x'/m_{px} represents the ratio between the accuracies of co-ordinate and parallax measurement in the photo. It varies according to the measuring device from $1\frac{1}{2}$ or 2 to 5. Since the y component of motion cannot be determined by this method, it is necessary to know the angle between direction of motion and direction of photography to obtain the horizontal motion ΔD. This makes it advisable to combine Methods A and B to determine velocities of glaciers.

Both methods can be used in principle with

---

[1] The formulae listed in Figure 3 are restricted to the normal case.

1. Normal Photography



in which : A, B = exposure station , P = any point
b = base
f = principal distance
x' z' = left photocoordinates of point
x" z" = right photocoordinates of point
px = x - parallax
x y z = ground coordinates of point referenced to the direction of the left photograph.

$$y = \frac{b \cdot f}{px} \qquad px = x' - x''$$

$$x = x' \frac{y}{f}$$

$$z = z' \frac{y}{f}$$

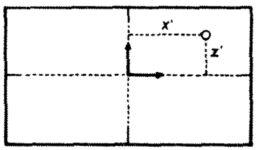
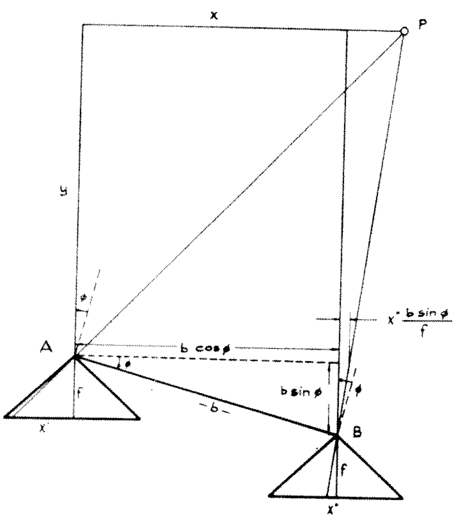


FIG. 1. Geometry of Terrestrial Photography:—*A*, Normal; *B*, Averted; and *C*, Convergent Terrestrial Photography.
B and *C* are on next page.

2. Averted Photography



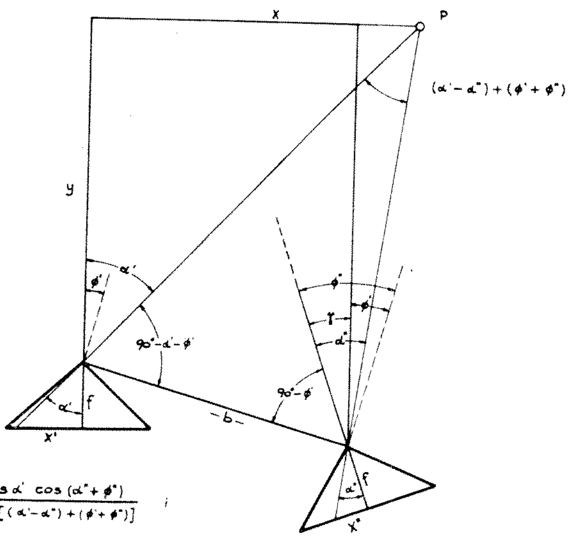
$$y = \frac{b\,(f \cos \phi + x'' \sin \phi)}{px} \quad ; \quad px = x' - x''$$

$$x = x' \frac{y}{f}$$

$$z = z' \frac{y}{f}$$

FIG. 1. Continued from page 66

3. Convergent Photography



$$y = \frac{b \cos \alpha' \cos(\alpha'' + \phi'')}{\sin[(\alpha' - \alpha'') + (\phi' + \phi'')]} \quad ;$$

$$\tan \alpha' = \frac{x'}{f}$$

$$\tan \alpha'' = \frac{x''}{f} \quad ;$$

$$x'' = x' - px \quad ; \qquad \gamma = \phi' - \phi''$$

$$x = x' \frac{y}{f}$$

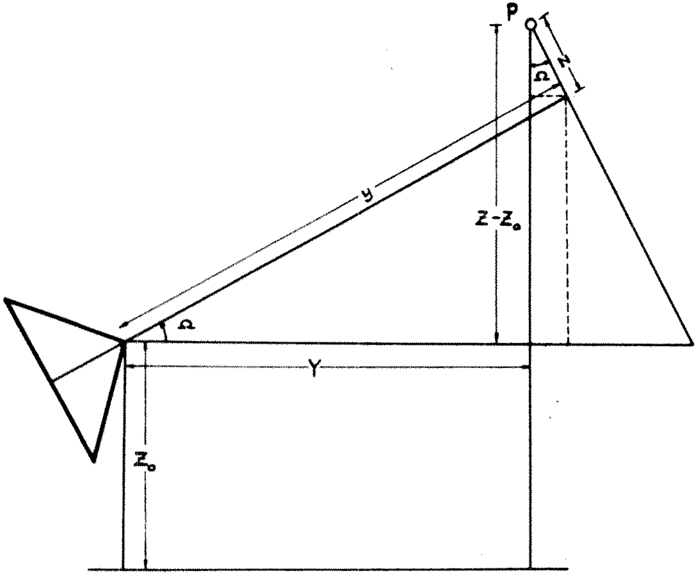
$$z = z' \frac{y}{f}$$

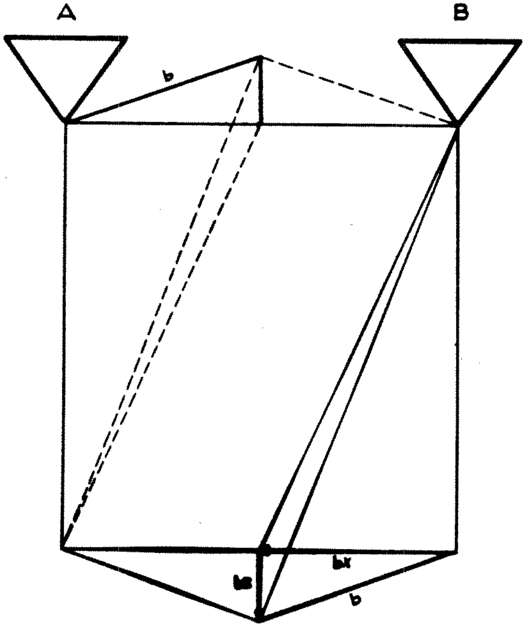
ϕ' is the aversion of the left camera and

ϕ'' of the right camera, both counted positive counterclockwise.

γ is the convergence, counted clockwise.

4. Inclined Photography





Y = y cos Ω − z sin Ω

Z−Z₀ = y sin Ω + z cos Ω

where : X , Y are groundcoordinates in a plane parallel to sea level.

Z is the height of the point

Z₀ is the height of the exposure station.

For the Wild A-5 or A-7 Autograph the inclination Ω is set as −ω in the plotter.

An aversion +ϕ is introduced by a −bz right.

γ can be set directly as a +φ right.

ω , φ , bz , refer to plotter readings

STANDARD ERROR

The co-ordinates of any identifiable point on the photograph can be determined with the following accuracy: (normal case)

$$m_y = \pm\sqrt{\frac{f^2}{px^2}m_b{}^2 + \frac{b^2}{px^2}m_f{}^2 + \frac{b^2f^2}{px^4}m_{px}{}^2}$$

$$m_x = \pm\sqrt{\frac{x'^2}{f^2}m_y{}^2 + \frac{y^2}{f^2}m_{x'}{}^2 + \frac{y^2x'^2}{f^2}m_f{}^2}$$

$$m_z = \pm\sqrt{\frac{z'^2}{f^2}m_y{}^2 + \frac{y^2}{f^2}m_{z'}{}^2 + \frac{y^2z'^2}{f^2}m_f{}^2}$$

in which: m_x, m_y, m_z, m_b, m_f, m_{px}, m_x', m_z' are the standard errors of:

x, y, z, b, f, px, x', and z'.

FIG. 2. The Use of Terrestrial Photography in First Order Plotters.

A. Stereoscopic Intersection Method

$$\Delta y = y_2 - y_1$$
$$\Delta x = x_2 - x_1$$
$$\Delta z = z_2 - z_1$$

The indexes 1 and 2 signify the exposures 1 and 2.

The differentials become:

$$d\Delta y = dy_2 - dy_1 = \frac{bf}{px^2}(dpx_1 - dpx_2) + \frac{b}{px}(df_2 - df_1) + \frac{f}{px}(db_2 - db_1)$$

$$d\Delta x = dx_2 - dx_1 = \frac{x'}{f}(dy_2 - dy_1) + \frac{yx'}{f}(df_2 - df_1) + \frac{y}{f}(dx'_2 - dx'_1)$$

$$d\Delta z = dz_2 - dz_1 = \frac{z'}{f}(dy_2 - dy_1) + \frac{yz'}{f}(df_2 - df_1) + \frac{y}{f}(dz'_2 - dz'_1)$$

Since the same base and camera are used $df_1 = df_2$ and $db_1 = db_2$

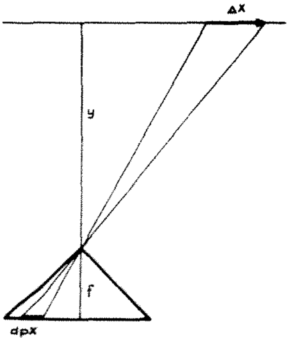
Thus:

$$m_{\Delta y} = \frac{bf\sqrt{2}}{px^2}\, m_{px}$$

$$m_{\Delta x} = \frac{y\sqrt{2}}{f}\, m_{x'}$$

$$m_{\Delta z} = \frac{y\sqrt{2}}{f}\, m_{z'}$$

B. Motion Parallax Method



$$\Delta x = \frac{y}{f}(x'' - x') = \frac{y}{f}\, dpx$$

$$\Delta z = \frac{y}{f}(z_2 - z_1) = \frac{y}{f}\, dpz$$

Δx and Δz can be determined with the following accuracy: (considering that $\frac{px}{f}, \frac{pxy}{f^2} \ll \frac{y}{f}$)

$$m_{\Delta x} = \frac{y}{f}\, m_{px}$$

$$m_{\Delta z} = \frac{y}{f}\, m_{pz}$$

The horizontal motion ΔD can be represented by:

$$\Delta D = \frac{\Delta x}{\cos\theta}$$

where $(90° - \theta)$ is the angle between direction of photography and direction of flow.

FIG. 3A, 3B Methods to Determine Motion from Terrestrial Photographs (In 2 parts.)

plaques or other signals. They are, however, mostly applied using natural points, making it unnecessary to have access to the glacier surface itself. Such an application, however, permits motion studies only for short periods since new snowfall or excessive melting may change the glacier surface to such an extent that identifying the selected natural detail points becomes impossible.

More accurate and more reliable than the selection of natural points is the use of a needle or a marking device such as the Zeiss Snap-Marker to prick a number of points selected along a glacier profile in one photograph, say $A(1)$, which will be used in conjunction with photo $B(1)$ to determine their position and with $A(2)$ to determine their movement stereoscopically.

c) AERIAL PHOTOGRAMMETRY

Aerial photogrammetry can be applied analogously to the methods of terrestrial photogrammetry. The disadvantage is that the exposure stations and the exposure directions are not recoverable in general.

Method A thus becomes a problem of mapping natural detailed points or signals to common ground control for photos $A(1)$, $B(1)$ and $A(2)$, $B(2)$, which must only be known approximately. The horizontal motion ΔD and the vertical motion ΔZ can be calculated from model co-ordinate differences between models (1) and (2).

Method B can also be applied. But the parallaxes measured will be affected by elevation differences, since the exposure stations of photos $A(1)$ and $A(2)$ will form a base b'; they will further be influenced by tilt. These influences will have to be separated from the observed parallax to determine the motion of the glacier.

Assuming near vertical photographs the method has been applied successfully by W. Hofmann (1958, pp. 71–88) to the glaciers of Greenland. The method is outlined in Figure 4:

It only neglects differences of second-order so that accuracy is mainly affected by the chosen flying height.

3. METHODS OF DETERMINING GLACIAL ADVANCE OR RETREAT

Glacial fluctuations can best be noticed by observing the advance or retreat of the glacier toe. The behaviour of the toe is influenced by various factors, most of all the topography and size of the glacier, so that volume changes provide more reliable data to interpret glacial advance or retreat.

Volume changes are best determined from contours. Glacier fluctuation studies thus require accurate contour mapping of the ice surface. While high contouring accuracy (say ±1 ft.) is normally desired, it is nevertheless sufficient to use a fairly large contour interval (say 100 ft.). This will cause a significant reduction in data and still will not introduce significant errors, since glacial volume changes are quite regularly distributed.

While mapping by ground survey methods is unsuitable except for toe surveys, both aerial and terrestrial photogrammetry will provide satisfactory results. Ground control requirements are not too critical, as long as the maps of the glacier produced at various intervals are based on the same control.

Methods to determine quantities significant for glacial fluctuations have been developed by R. Finsterwalder (1953, pp. 189–239). (See Figure 5.)

Their accuracy will depend on the error of contouring, as well as on the accuracy of area determination (planimeter or grid square counting).

It should be noted that the error of a contour is influenced by the terrain slope. (See Figure 6.) (Finsterwalder 1952; 1957, pp. 1–7.)

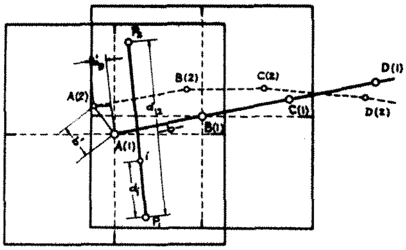
In determining volume and height changes areas are used; thus the position error m_p' of the contour will reflect the accuracy.

This is of importance when comparing accuracies of terrestrial and aerial photogrammetric methods, since often the position errors of a contour (m_p') may be of equal magnitude for both methods, while terrestrial photogrammetry may be at a disadvantage as far as the height error m_h' is concerned. For glacial volume studies, however, this is immaterial. The height and position errors of measurement m_h and m_p causing the contour errors will naturally be dependent on flying-height or base-length respectively (see Figure 1).

4. GLACIAL SURVEYS IN WESTERN CANADA

The methods outlined have been used on various glaciers in the West of Canada in the past. (See Figure 7.)

The Department of Northern Affairs and National Resources (Water Resources Branch), has conducted a periodic plaque line and toe stadia survey observation program for several glaciers since 1945 (1945, 1946, 1947, 1949, 1950, 1952, 1956, 1960, 1961, 1962).



C. AERIAL PHOTOGRAMMETRY

Parallaxes px between photos $A(1)$ and $A(2)$ are measured along a chosen speed profile P_1 and P_2. P_1 and P_2 are points situated on rock close to the edges of the glacier. The parallaxes are influenced by tilt and height relief due to base b' between $A(1)$ and $A(2)$. The necessary reductions become:

1. ***Reduction due to tilt of line*** P_1P_2.

$$dpx_i = \frac{d_i}{d_{1.2}}(px_{P2} - px_{P1}) \equiv \text{reduction}$$

$$\Delta px_i = px_i - dpx_i - px_{P1} \equiv \text{relative parallax}$$

px_i, px_{P1}, px_{P2}, are the absolute parallaxes of points i, P_1, P_2, measured between photos $A(1)$ and $A(2)$; $d_{1.2}$ and d_i are the distances between points 1 and 2 or 1 and i.

2. ***Reduction due to height.***

Height differences Δh are determined by measurement of parallaxes Δp_i using photos $A(1)$ and $B(1)$ forming base b. The same height differences Δh will cause a reduction δpx_i due to base b'.

$$\frac{h^2}{bf}\Delta p_i = \Delta h_i = \frac{h^2}{b'_D f}\delta px_i$$

b'_D is the component of b' perpendicular to profile P_1P_2.

$$\delta px_i = \frac{b'_D}{b}\Delta p_i \equiv \text{reduction}$$

The *horizontal motion* ΔD thus becomes:

$$\Delta D = \frac{h}{f}(\Delta px_i - \delta px_i)\cos\Theta$$

Θ is the angle between profile and the normal to the direction of motion.

FIG. 4. Methods to Determine Motion from Aerial Photographs.

A detailed survey of the Saskatchewan Glacier by similar methods was made by the U. S. Geological Survey from 1952 to 1954. (Meier, 1960).

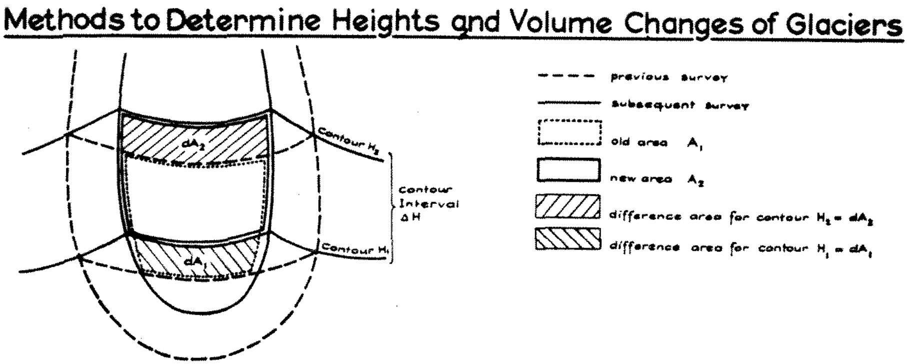
Aerial and partially terrestrial photogrammetric methods were introduced by the National Research Council of Canada in 1957, while mapping the Salmon Glacier (Haumann, pp. 74–110).

A detailed aerial photogrammetric survey of the Athabaska glacier was made in 1959 by the Department of Northern Affairs and National Resources. It was repeated in 1962 (1959).

The information obtained by these surveys is still scarce so that comparisons of glacial behaviour, as they were done in the Alps (Finsterwalder, 1953, pp. 189–239), are not yet fully possible.

In the meantime it is desirable to extend the observation program to other glaciers, so far unsurveyed, and to continue regular surveys at periodic intervals for the glaciers already surveyed, and this by the best possible methods.

It is under these aspects that a research project was undertaken by the University of New Brunswick. The objectives were to provide glaciological data for a few glaciers of different size and type in the area of Mt. Athabaska in Alberta. Terrestrial photogrammetry was chosen as the method of survey, since only limited funds were available. The area was selected because of its accessibility



1. Height change dh_i for contour interval $(H_2 - H_1)_i = \Delta H$

$$dh_i = \frac{dA_1 + dA_2}{A_1 + A_2} \Delta H$$

2. Volume change dV_i for contour interval $(H_2 - H_1)_i$

$$dV_i = \frac{dA_1 + dA_2 + \sqrt{dA_1 \, dA_2}}{3} \Delta H$$

3. Mean Height change dh_m for the glacier per year

$$dh_m = \frac{\Sigma dV_i}{A_{\text{old}} - A_{\text{new}}} \cdot \frac{2}{n}$$

where A_{old}, AV7 are the total glacier areas used for the summation of dV_i and n is the number of years between the surveys.

FIG. 5. Methods to Determine Heights and Volume Changes of Glaciers.

and also because plaque line and aerial photogrammetric surveys had already been conducted at one of the glaciers of the area, the Athabaska Glacier. This would allow a welcome comparison for the various survey methods.

5. ATHABASKA GLACIER SURVEY

The terrestrial photogrammetric survey covered 8 glaciers, the two largest of these being the-Saskatchewan and the Athabaska glaciers. (See Figure 8.) The evaluation of the data is still in progress so that only comparisons for the Athabaska glacier can be made here:

A. VELOCITY DETERMINATIONS

A velocity profile was determined from a base alongside the glacier. The interval of photography was two and four days. Results of the measurements made on the Jena 1818 stereo comparator and their standard errors are given in Table 1. These (96 feet per year) agree quite well with average velocities of 106

ACCURACY OF PHOTOGRAMMETRIC CONTOURS

The accuracy of a contour depends upon the terrain slope α. An error may be caused by an error in height measurement m_h or by an error in position m_p.
From these result:
a height error of the contour

$$m_h' = \sqrt{m_h^2 + m_p^2 \cdot \tan^2 \alpha}.$$

and a position error of the contour

$$m_p' = \sqrt{m_h^2 \cot^2 \alpha + m_p^2}$$

FIG. 6. Accuracy of Photogrammetric Contours.

ft. and 62 ft. per year measured along the two plaque lines of the Water Resources branch (1945, 1946, 1947, 1949, 1950, 1952, 1956, 1960, 1961, 1962). No photography was available which would have permitted a velocity determination by aerial photogrammetry.

B. DETERMINATION OF VOLUME CHANGES

A direct comparison of volume changes can be made between the aerial photogrammetric surveys conducted by the Department of Northern Affairs and National Resources and the terrestrial photogrammetric survey made by the University of New Brunswick:

a) *Photography*

Figure 9 shows four phototheodolite photographs. It will be noted that the glacier area appears small in comparison to the details visible in the aerial photographs shown on Figure 10. From the point of view of interpretation the aerial photographs are certainly more satisfactory. For aspects of measurement this is not as important if the required intersection accuracy can be ascertained for the contours.

A distinct disadvantage of terrestrial photographs is that sometimes the survey area cannot be seen entirely from a suitable base. Gaps may occur which have to be filled by taking photographs from additional base lines. These may make terrestrial surveys under such adverse conditions time-consuming, costly and treacherous because of the weight of the equipment to be carried around. The Mt. Athabaska area did not present such difficulties. It can be considered generally suitable for terrestrial photogrammetry. Under these circumstances the taking of the photographs can be done at negligible cost, since it can be combined with the ground control survey, while aerial photographs in mountains may become expensive because of bad weather waiting periods.

b) *Survey Accuracies*

Table 2 lists the particulars under which survey photography was taken for both methods. From these, theoretical accuracies are computed for the practical case of using aerial and terrestrial contour maps plotted on the Wild A7 and A5 plotters respectively. It will be noted that under the given conditions the terrestrial survey yields higher accuracy in the toe area, while the firn area is more accurately mapped by the aerial survey. This is due to the increase of error with the square of the object distance in terrestrial photogrammetry. Both methods, however, should give comparable results.

TABLE 1

VELOCITY PROFILE, ATHABASKA GLACIER

Point	*Photos:* *Interval:*	$A(1)\ A(3)$ 4 *Days* ΔD_{13} (*meters*)	$A(1)\ A(2)$ 2 *Days* ΔD_{12} (*meters*)	$A(2)\ A(3)$ 2 *Days* ΔD_{23} (*meters*)
1		0.23	0.09	0.19
2		0.23	0.12	0.18
3		0.16	0.09	0.07
4		0.32	0.16	0.16
5		0.24	0.19	0.15
6		0.35	0.26	0.22
7		0.36	0.19	0.25
8		0.34	0.19	0.24
9		0.34	0.23	0.20
10		0.33	0.18	0.23
11		0.38	0.18	0.24
12		0.43	0.15	0.30
13		0.35	0.17	0.24
14		0.34	0.18	0.24
15		0.44	0.19	0.28
16		0.33	0.16	0.22
17		0.31	0.13	0.18
18		0.33	0.15	0.19
19		0.33	0.17	0.20
20		0.31	0.12	0.16
Average:		0.32 m	0.16 m	0.21 m

The standard error of one observation as computed from the above measurements is ±0.06 m.

An extrapolation (if permissible) would give a velocity of 96 ft. per year.

c) *Ground Control*

Figure 11 shows the ground control needed for the projects. Terrestrial photogrammetry with about five control points is clearly at an advantage over the 22 control points of the aerial survey. It should be further noted that control for terrestrial photogrammetry is only used to correct errors of known interior or exterior orientation, but not to actually control the model.

It will be noticed that the ground control required for aerial photography has rigid limitations as to location. This will necessitate the use of ground control points situated on moraines or debris, which cannot be considered permanent markers for resurveys. Table 3 illustrates this by listing the considerable movements of some of the control plugs, as determined by the Department of Northern Affairs for the 1959 and 1962 aerial surveys.

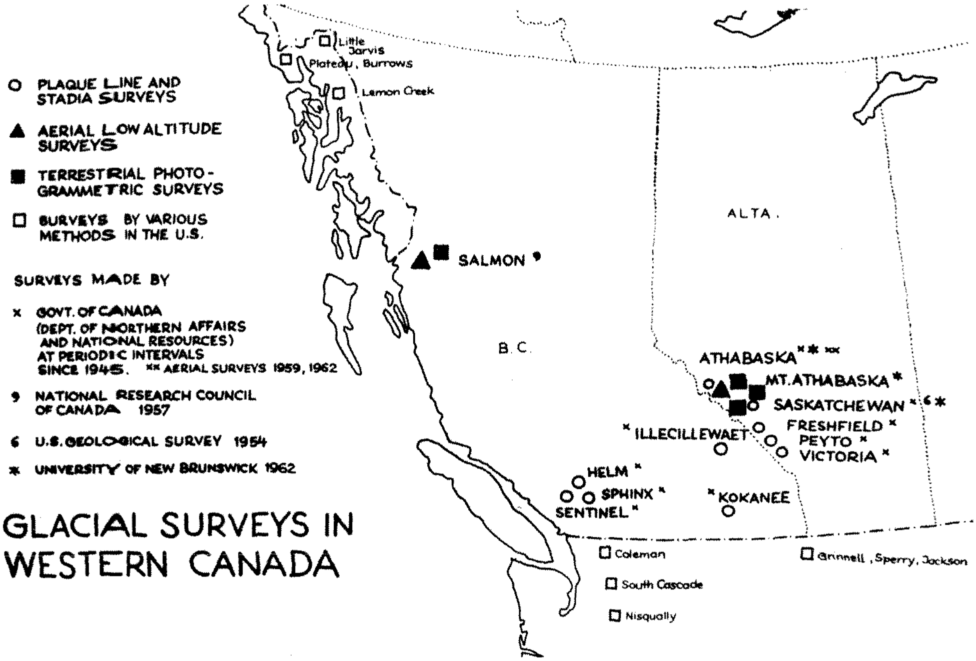


FIG. 7. Glacial Surveys in Western Canada.

For terrestrial surveys, exposure stations and control can almost always be chosen on rock, so that resurveys at the time of rephotography do not become necessary.

Figure 12 shows that ground control required for terrestrial photogrammetry lends itself more easily to triangulation procedures, so that by observing one net, as was done for the Mt. Athabaska area, several glaciers can be controlled at the same time with a minimum of additional effort. This entire survey was possible during a season of one month, while a considerably longer period of time was needed for the aerial survey control. The use of bridging procedures would make aerial surveys of glaciers more economical, but also more complex.

d) *Plotting*

The plotting of terrestrial photographs on a first-order plotter may encounter the difficulty of having to use varying model scales to accommodate the distance range due to the Z-limitations of the instrument. Plotting can, however, be done in the same scale by changing the gears between autograph and drawing table. On the other hand, orientation of terrestrial photos is much faster than for aerial pairs if the exposure directions and the levels of the phototheodolite have been set carefully at the time of the photography. Control serves then merely as a check for preset instrument settings.

e) *Results*

While a detailed comparison of the methods used in 1962 is still underway, using contour testing procedures (Finsterwalder 1957, pp. 1–7), volume changes have been calculated by comparing contours of surveys in 1962, 1959, and 1919. The survey of 1919 cannot be used for accurate computations, since it constitutes part of a comparatively not-too-accurate map compiled in connection with the British Columbia-Alberta boundary survey. Historic records are nevertheless of interest. Figure 12 shows the contours superimposed, which were used for calculations.

Table 4 gives the calculated height and volume changes for each contour interval. It indicates that the glacier has had an average loss in height of 5.6 ft., and a loss in volume of 13.9 million cubic feet during the three year period. A comparison of the values with volumes determined by stadia surveys in the toe area (1945, 1946, 1947, 1949, 1950, 1952, 1956, 1960, 1961, 1962) is not directly possible for the 3 year period. But the maps compiled

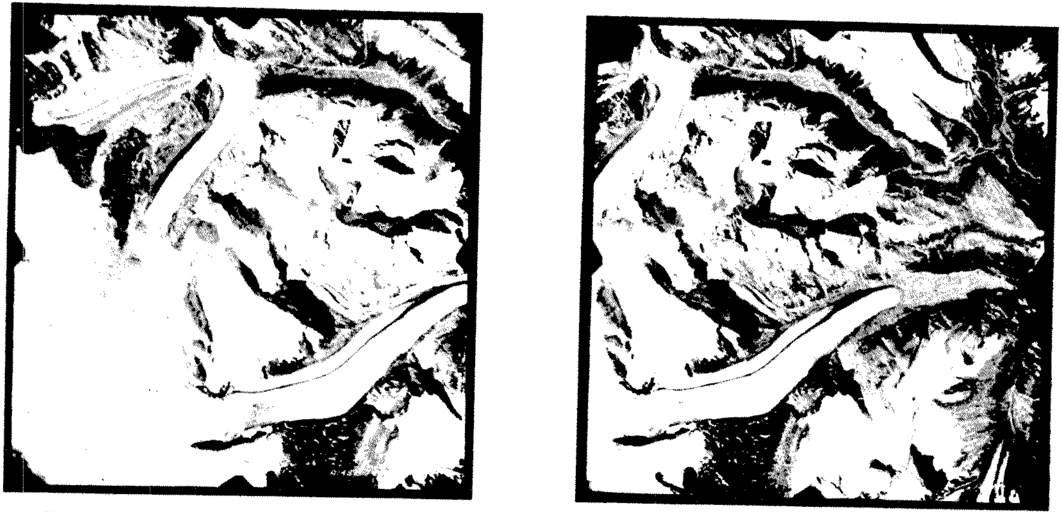


Fig. 8A. Columbia Icefield, Dome Glacier, Athabaska Glacier, Mt. Athabaska.

Fig. 8B. Mt. Athabaska, Saskatchewan Glacier.

Fig. 8. Aerial Photography from 30,000 feet altitude above ground. Survey Area, Mt. Athabaska.

	AERIAL PHOTOGRAMMETRY	TERRESTRIAL PHOTOGRAMMETRY	*Toe (front)*	*Firn Area (back)*
	$h = 6{,}000$ ft.	$y = 1$–3.5 miles (6,000–18,000 ft.)		
	$b = 2{,}400$ ft.	$b = 2{,}400$ ft.		
	$f = 152$ mm	$f = 165$ mm		
Point accuracy	$m_z = \pm \frac{h^2}{bf} m_{px} = \pm 0.7$ ft.	$m_z = \pm \frac{y}{f} m_z'$	$= \pm 0.4$ ft. to	± 1.2 ft.
	$m_x = \pm \frac{h}{f} m_x' = \pm 0.4$ ft.	$m_x = \pm \frac{y}{f} m_x'$	$= \pm 0.4$ ft. to	± 1.2 ft.
	$m_y = \pm \frac{h}{f} m_y' = \pm 0.4$ ft.	$m_y = \pm \frac{y^2}{bf} m_{px}$	$= \pm 0.6$ ft. to	± 5.7 ft.
	$m_p = \pm \sqrt{m_x^2 + m_y^2} = \pm 0.6$ ft.	$m_p = \pm \sqrt{m_x^2 + m_y^2}$	$= \pm 0.7$ ft. to	± 5.7 ft.
	for $m_{px} = 7\mu$	for $m_{px} = 7\mu$		
	$m_x' = m_y' = 10\mu$	$m_x' = m_z' = 10\mu$		
Contour accuracy (position)	$m_p' = \sqrt{m_z^2 \cot^2\alpha + m_p^2} =$	$m_p' = \sqrt{m_z^2 \cot^2\alpha + m_p^2} =$		
	for $\alpha = 14°$ (Toe) ± 2.9 ft.	± 1.7 ft. (to ± 7.5 ft.)		
	for $\alpha = 3°$ (Firn) ± 15.4 ft.	(± 8.8 ft.) to ± 27.2 ft.		
Volume accuracy (for height zone $\Delta H = 100'$)	$m_{dV} = \pm \frac{\Delta H . w . \sqrt{2}}{2}$	$m_{dV} = \pm 0.3 \times 10^6$ to $\pm 4.6 \times 10^6$ cu. ft.		
	$m_p' = 0.5 \times 10^6$ to $\pm 2.6 \times 10^6$ cu. ft.			

w = width of glacier, ≈ 2400 ft.

TABLE 3

MOVEMENTS OF GROUND CONTROL POINTS FROM 1959 TO 1962 (1959 POSITION)-(1962 POSITION)

	d (elevation) ft.	*dE (metres)*	*dN (metres)*	*Total horizontal movement (metres)*
2	+ 0.03	—	—	—
3	+ 0.03	—	—	—
4	− 0.03	+ 0.03	+0.09	0.09
5	+ 0.14	+ 0.09	−0.04	0.09
6	− 0.09	− 0.10	—	0.10
7	+ 0.39	+ 0.15	−0.04	0.15
8	− 0.34	+ 0.03	—	0.03
9[1]				
10	− 0.27	+ 0.07	−0.04	0.08
11	+ 0.43	+ 0.73	+0.03	0.74
12[2]	+23.69	−11.89	−3.60	12.41
13	− 0.27	+ 0.16	+0.06	0.17
14	+ 0.40	− 0.17	+0.09	0.19
15	− 0.34	+ 0.14	—	0.14
16	− 0.32	+ 0.15	−0.03	0.15
17[2]	+ 7.51	+ 0.09	−5.37	5.38
18	− 0.26	+ 0.23	—	0.23
19	− 0.34	+ 0.17	+0.06	0.18
20	− 0.30	+ 0.24	—	0.24
21	− 0.27	+ 0.20	+0.06	0.21

[1] New station set up in 1962.
[2] Significant change in position.

TABLE 4

HEIGHT AND VOLUME CHANGES FOR DIFFERENT HEIGHT ZONES OF THE ATHABASKA GLACIER FROM 1959 TO 1962

Height Zone (ft.)	*dh (ft.)*	*dV 10^6 (cu. ft.)*	*Area 10^4 (sq. ft.)*
6320–6400	−24.8	−11.01	45.60
6400–6500	−29.1	−16.32	56.00
6500–6600	−22.6	−16.00	71.04
6600–6700	−13.6	−14.72	108.48
6700–6800	− 7.6	−12.48	163.36
6800–6900	− 3.9	− 9.60	250.24
6900–7000	− 7.9	−21.60	274.56
7000–7100	− 7.0	−17.28	246.72
7100–7200	− 2.8	−10.88	384.64
7200–7300	+ 1.1	+ 7.36	662.56
7300–7400	+ 1.5	+ 7.84	536.16
7400–7500	− 0.6	− 2.24	368.32
7500–7600	+ 3.0	+ 3.68	121.92
7600–7700	+ 5.9	+ 8.32	141.76
7700–7800	+ 9.2	+26.88	292.16
7800–8000	+18.0	+60.48	336.00
8000–8200	+ 6.2	+34.08	551.20
8200–8400	− 3.5	− 7.84	225.12
8400–8600	−28.7	−22.56	78.72
Total	−107.2	−13.89	4914.56
Mean	− 5.64		
Mean per year	− 1.88	− 4.63	

Fig. 9A. NORTHERN GLACIER OF MT. ATHABASKA From Chalet Ridge

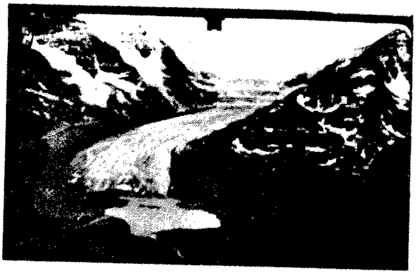


Fig. 9B. ATHABASKA GLACIER From Chalet Ridge

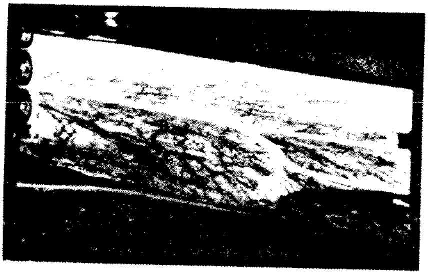


Fig. 9C. VELOCITY PROFILE PHOTOGRAPH From Base "snowmobile"

Fig. 9D. EASTERN GLACIER OF MT. ANDROMEDA From Base "Moraine"

Fig. 9. Terrestrial Photography 1962.

Fig. 10. Aerial Photography from 6000 ft. altitude above ground, Athabaska Glacier 1939. (Firn Area at left. Toe area at right.)

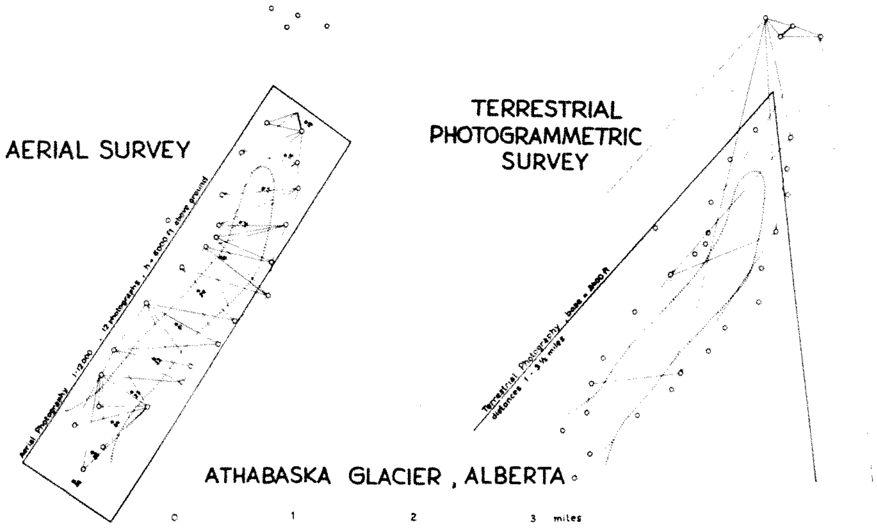


FIG. 11. Comparison of Coverage and Ground Control for Aerial and Terrestrial Photography.

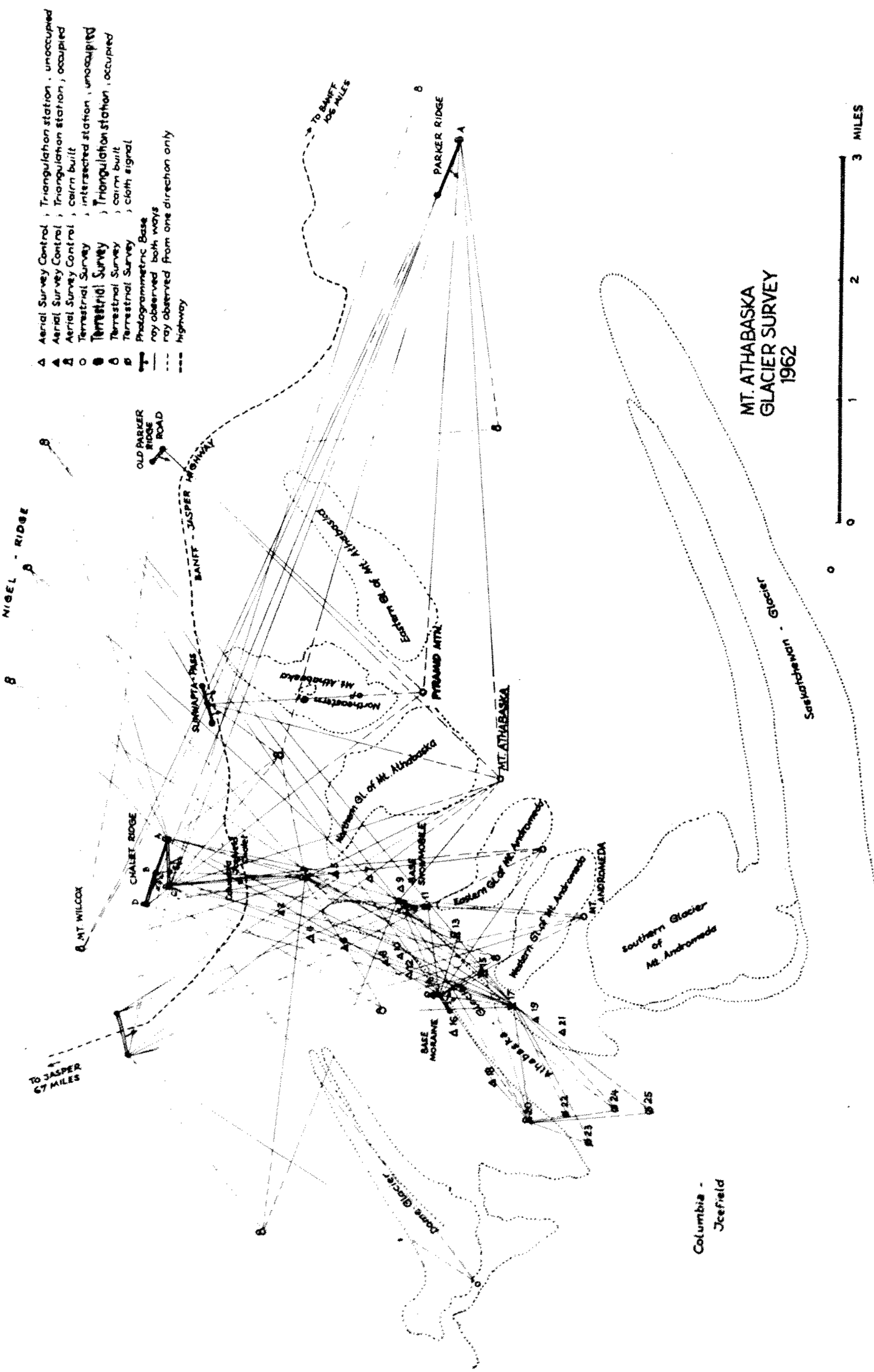


FIG. 12. Mt. Athabaska Glacier Survey Triangulation Net 1962.

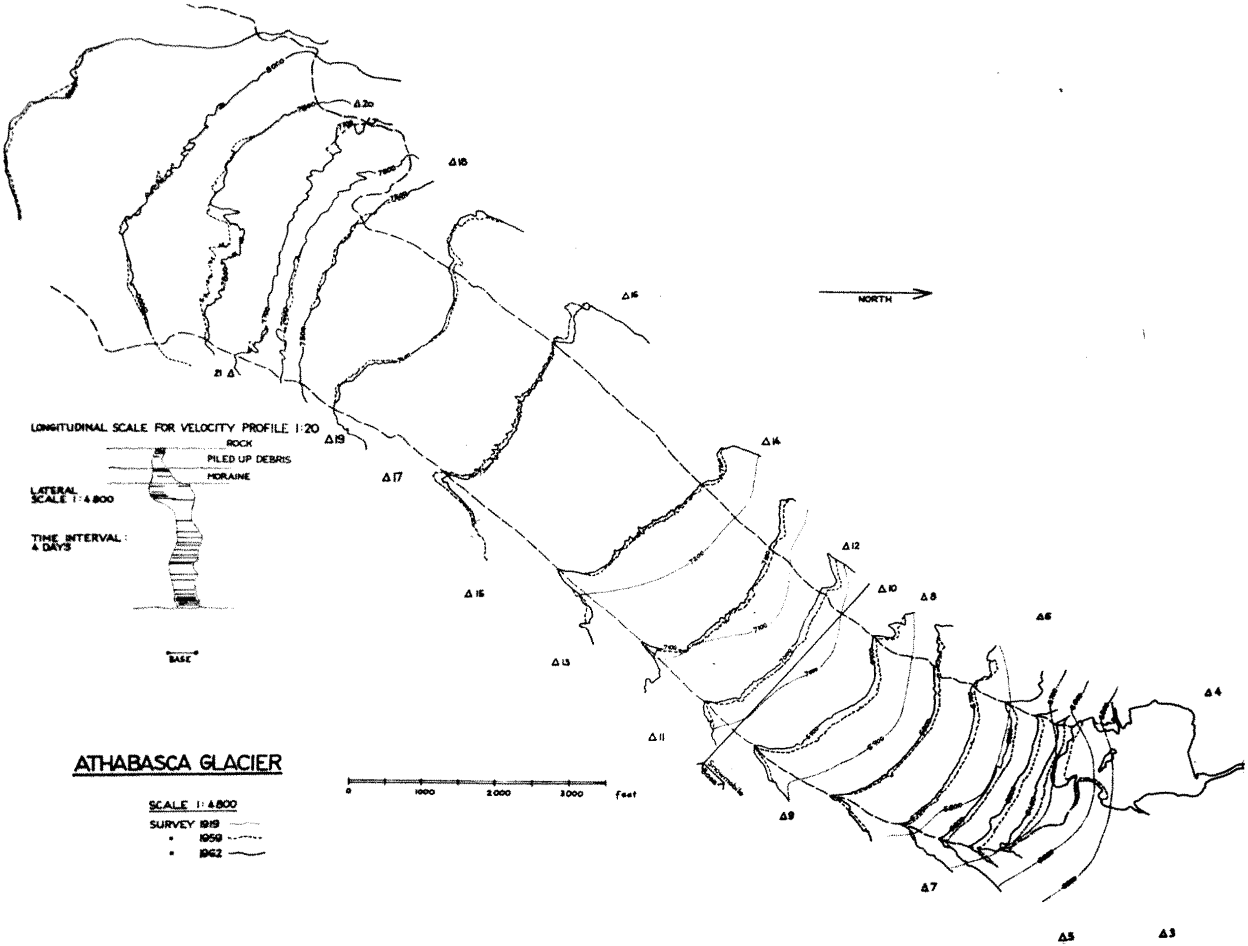


FIG. 13. Retreat of the Athabaska Glacier between 1919, 1959 and 1962.

for 1962 will permit a comparison between results obtained by stadia, aerial and terrestrial photogrammetry. These will be presented in a final report when the project is completed.

6. CONCLUSIONS

It has been shown that each method used for glacial surveys has its particular advantages.

It is not the purpose of this paper to suggest which method will be best for application in glacial studies, since the comparison made may not be characteristic for other areas. The Mt. Athabaska area has relatively small glaciers; this may make it more suitable for terrestrial photogrammetric procedures than others. On the other hand, the small glaciers are of greatest interest, since the response to climatological changes is the greatest. Glacial surveys are not necessarily synonymous with glacial mapping. All survey methods should be employed in conjunction to obtain the best possible survey result.

ACKNOWLEDGMENTS

Acknowledgment is made to the National Research Council which provided the grant funds for conducting the terrestrial photogrammetric surveys; to the Department of Northern Affairs and National Resources, which was very co-operative in providing all available information about the previous and concurrent aerial and stadia surveys; to Mr. I. A. Reid of the Water Resources Branch, who gave valuable advice and aid during the survey; and to the Topographical Survey of Canada, which assisted in the project by the loan of mountaineering equipment.

REFERENCES

1. Blachut, T. J., "The Use of Aerial Photogrammetry in the Study of Mountain Glaciers." Publ. No. 54 of I.A.S.H., Snow and Ice, pp. 484–492.
2. Case, J. B., "Mapping of Glaciers in Alaska." PHOTOGRAMMETRIC ENGINEERING 1958, pp. 815–821.
3. "Glacier Mapping in the Western United States." Ohio State University Research Foundation, Report 943, January 1960.
4. Colcord, J. E., "The TAF Phototheodolite and its Use in Glacier Surveys." PHOTOGRAMMETRIC ENGINEERING 1957, pp. 552–557.
5. Department of Northern Affairs and National Resources, Water Resources Branch, Ottawa. "Triangulation Survey of the Athabaska Glacier," July 1959, I. A. Reid.
6. Glacial Survey Reports 1945, 1946, 1947, 1949, 1950, 1952, 1956, 1960, 1961, 1962, by McFarlane, May, Campbell and Davis.
7. Finsterwalder, R., "Geschwindigkeitsmessungen an Gletschern mittels Photogrammetrie," Zeitschrift für Gletscherkunde, Innsbruck 1931, pp. 252–263.
8. "Photogrammetrie," De Gruyter, Berlin 1952.
9. "Die zahlenmässige Erfassung des Gletscherrückgangs an Ostalpengletschern," Zeitschrift für Gletscherkunde, Innsbruck 1953, pp. 189–239.
10. "Photogrammetrische Höhenschichtlinien" Zeitschrift für Vermessungswesen, 1957, pp. 1–7.
11. "Scope, State and Development of Precise Glacier Surveys on the Earth." General Assembly IUGG, Toronto 1957, Vol. IV, pp. 520–524.
12. "Measurement of Ice Velocity by Air Photogrammetry." Symposium Chamonix, IUGG, 1958, pp. 11–12.
13. "On the Measurement of Glacier Fluctuations," Pub. No. 54 of I.A.S.H., *Snow and Ice*, pp. 325–334.
14. Haumann, D. "Photogrammetric and Glaciological Studies of Salmon Glacier," *Arctic*, Montreal, Vol. 13, No. 2, pp. 74–110.
15. "Luftphotogrammetrische Gletschervermessungen in Kanada," *Zeitschrift für Vermessungswesen*, Stuttgart 1960, pp. 388–397.
16. Hofmann, W., "Bestimmung von Gletschergeschwindigkeiten aus Luftbildern," *Bildmessung und Luftbildwesen*, Berlin 1958, pp. 71–88.
17. "Der Vorstoss des Nisqually Gletschers am Mt. Rainier, U.S.A., von 1952 bis 1956," *Zeitschrift für Gletscherkunde*, Innsbruck, 1958, pp. 47–60.
18. Meier, M. F., "Mode of Flow of Saskatchewan Glacier, Alberta, Canada," U. S. Geological Survey Paper 351, Washington, 1960.
19. Zeller, M., "Lehrbuch der Photogrammetrie," Orell Füssli, Zürich 1947.