Phenomena Associated with the Deformation of a Glacier Bore-hole Taku Glacier, Alaska

by

Maynard M. Miller
Lamont Geological Observatory
Columbia University

Extrait des Comptes Rendus et Rapports -- Assemblée Générale de Toronto 1957
(Gentbrugge 1958)
Tome IV, p.437 à 452

Extrait des Comptes Rendus et Rapports — Assemblée Générale de Toronto 1957
(Gentbrugge 1958)
Tome IV, p. 437 à 452

PHENOMENA ASSOCIATED WITH THE DEFORMATION OF A GLACIER BORE-HOLE

MAYNARD M. MILLER
Lamont Geological Observatory, Columbia University

Abstract

A stress/strain analysis is made of the deformation of a 2-inch (I. D.), 245-foot length of jointed aluminum tubing inserted in a vertical bore-hole in the Taku Glacier of the Juneau Icefield, Alaska. The data embrace three years of consecutive measurement of the changes in azimuth and inclination of the tube in response to the glacier's flow. The results are related to Glen's and Steinemann's exponential flow equations, as derived from laboratory tests on polycrystalline ice, and are compared with bore-hole and tunnel experiments conducted in glaciers of the Alps and Norway. The total depth of ice is known by seismic means. Although the tube penetrates only the upper one-quarter of the Taku Glacier, it is demonstrated that the periodic surveys permit a reasonable extrapolation of the differential flow velocity on a vertical line completely through the glacier. It is shown that with the internal movement dominated by *continuous* plastic creep, a supplemental longitudinal stress trajectory has been superimposed on the normal shear component of flow. In consequence, the low-gradient sectors of the glacier are characterized by a high proportion of basal slippage. This concept is corroborated by the «Block-Schollen» surface velocity profiles and by the existence of tectonic foliation and thrust structures in a narrow zone of *discontinuous* movement near the glacier's margin. Certain anomalies in the bore-hole deformation (creep rate) are explained by configuration of the bedrock floor, and by measured physical differences in the structure of the glacier itself. Observed thermal anomalies on the profile are also considered. These are found to coincide with deformation anomalies and to indicate that zones of inhibited flow occur where colder englacial temperatures prevail.

Introduction

With a sufficiently high stressing condition the internal movement of a glacier can involve processes both of discontinuous shear and continuous flow (Haefeli, 1952). The extent to which these mechanisms co-exist in a large valley glacier has been investigated on the Taku-Llewellyn Glacier system in Southeastern Alaska (Fig. 1). The study has involved determination of profiles of surface and englacial velocity. This has been accomplished by the periodic survey of lines of stakes and other markers on the surface, and by the measurement of changes in inclination and azimuth of an aluminum tube implanted vertically in the glacier. The phenomena associated with the deformation of this tube are discussed in this paper.

Research Site and Techniques

The main research was carried out on a low-gradient portion of the Taku Glacier's névé near Camp 10 (Fig. 1), at a point 3 miles above the mean névé line and 16 miles above the terminus. The drill site (Station 10B, elev. 3600 feet) was on a horizontal movement profile, one mile out from the eastern margin in a sector where the névé was 4 miles wide and where the glacier's depth had been determined by seismic means to be approximately 1000 feet (Fig. 2). A rotary drill rig was used to bore

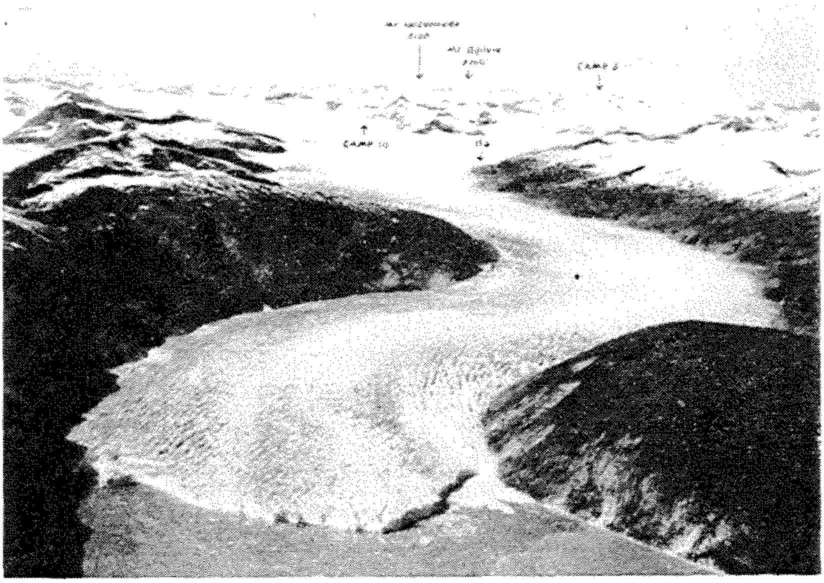


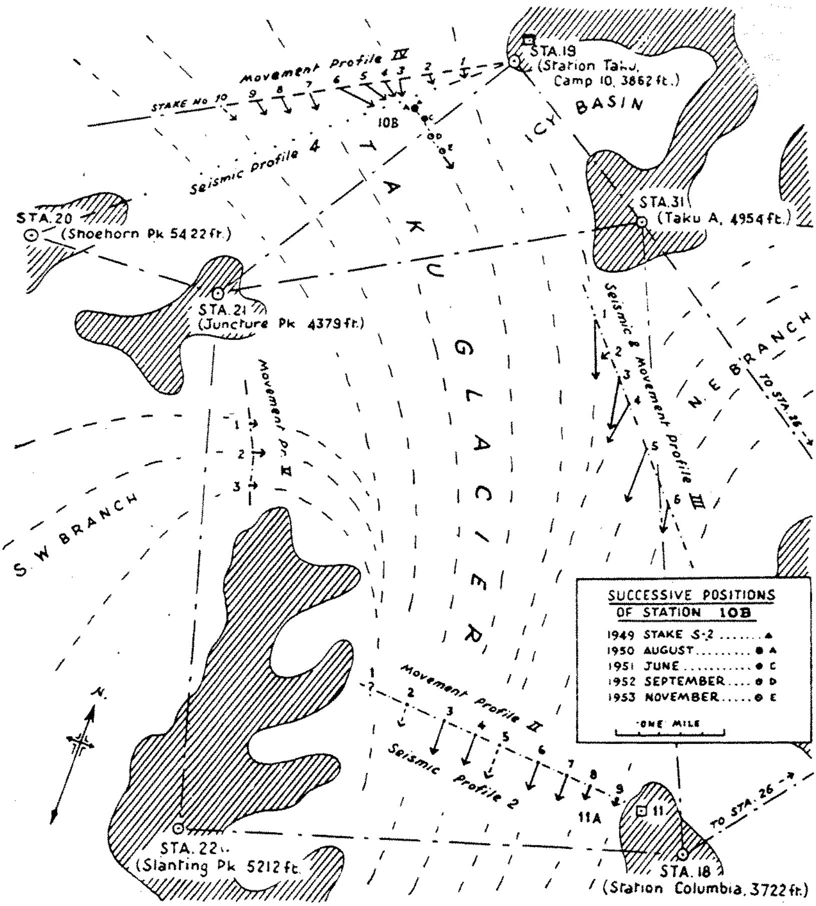
Fig. 1 — Oblique aerial view of Taku Glacier showing tidal terminus in 1929.

a hole 292 feet deep. A 2-inch aluminum pipe, in 10-foot jointed lengths, was inserted into the upper 245 feet of this bore-hole. (1)

Broad and relatively level upland basins comprise most of the accumulation area of this glacier system. Scattered nunataks account for only a small proportion of the surface within the main névé area. Thus there appears to be little opportunity for the ice to be confined or blocked in its movement at depth. A « Block-Schollen» type of flow profile is indicated both on the cross-glacier transect at the drill site (Movement Profile IV) and on another transect at 2600 feet elevation (Movement Profile II), 3 miles below the mean névé line (Fig. 3). On the margins of the glacier in the vicinity of Profile II (e.g. near Site 11A in Fig. 1), a sequence of closely-spaced tectonic foliation structures occurs in the ice, suggesting that the marginal zones of high «mobility» on each side of the glacier may be explained by *discontinuous* laminar movement.

In the central part of each transect, no appreciable discontinuous movement is indicated. This is corroborated in the bore-hole by the fact that in the core samples there was no evidence of over-thrusting. Also in these cores small-scale fracturing, such as that occurring in the marginal zones, appeared to be so restricted that it could account for little, if any, of the mass movement. On empirical grounds, therefore, uniform plastic yielding is considered as the all important mechanism of internal motion in the central part of this glacier. A similar conclusion was reached by the

(1) For this research, a Pioneer Straight-line drill rig was employed, loaned by the E. J. Longyear Co. of Minneapolis, Minn. The drilling equipment and accessories totalled 8 tons and were delivered to the research site by ski-aircraft supporting the Juneau Icefield Research Project (Miller, 1951).



1950 TO 1953 GLACIER SURFACE MOVEMENT SURVEYS IN VICINITY OF CAMPS 10 AND 11

Showing Direction and Relative Magnitude of Average Daily Movement across Key Depth Profiles During Months of July and August. Velocity Scale Exaggerated 500 X. (Nunatak outlines only approximate and base triangulation network diagrammatic)

Fig. 2

Jungfraujoch Research Party (Perutz, 1940, p. 134) on the Aletsch Glacier in Switzerland, an ice stream of similar size and form to the Taku.

Depth of Firn

During the drill operations all core samples were structurally analyzed as they were brought to the surface. A continuous sequence of firn was identified down to 120 feet. A relatively dense material (firn-ice?) was encountered below 85 feet and cores of what appeared to be mostly aerated bubbly ice were recovered between

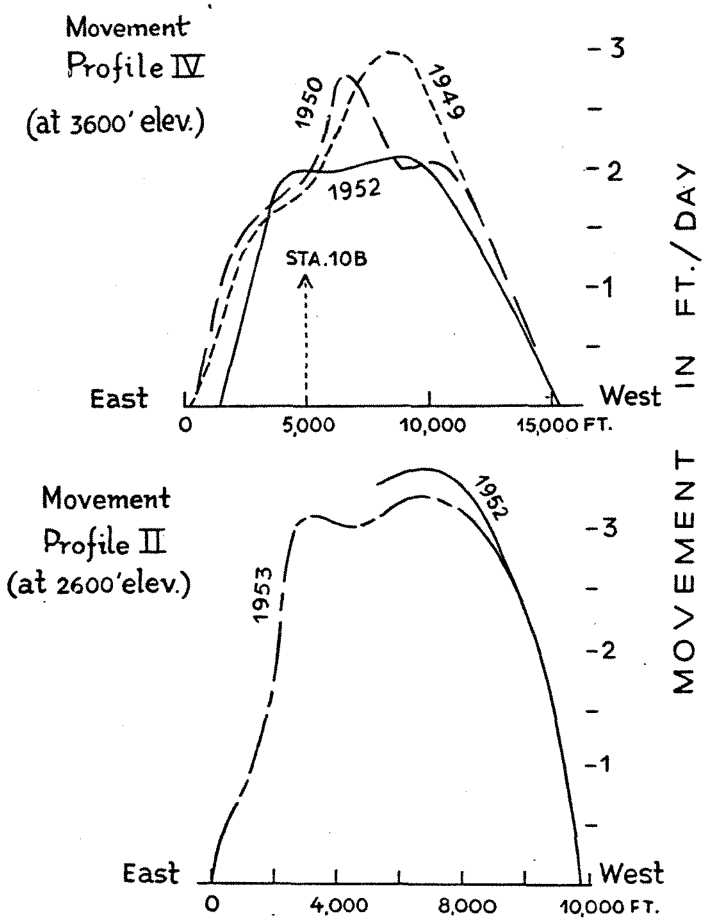


Fig. 3 — Surface velocities in the Névé-Line zone of the Taku Glacier.

120 and 140 feet. In this zone the cores were so crushed and distorted by the churning action of the drill that they weie difficult to assess. Below 140 feet, however, only solid ice was encountered. The thickness of firn is thus considered to be about 120 feet, with an unexpectedly abrupt change indicated between the firn-pack and deeper ice at this site (Fig. 4). Evidence from elsewhere on the icefield has suggested that this sudden change is related to a pronounced alteration of climatological conditions during the early decades of the last century (Miller, 1956, ch. XII, B4).

The Bore-hole Surveys

The interior of the aluminum tube was surveyed with a magnetic inclinometer manufactured by the Eastman Oil Well Survey Co. of Denvei, Colorado. The shot points were spaced 5- to 20-feet apart along the tube. Pertinent statistics over a

38-month period are noted in Table I. The first survey was in August 1950 and the last in November 1953. At this time, the protruding top of the pipe was found to be excessively tilted down-glacier and to have become constricted by buckling somewhere in the upper 50 feet so that the inclination could not be measured at depth. Details of the su veys, and plan and cross-sectioned plottings of the results, have been given in previous publication (Miller, 1954a).

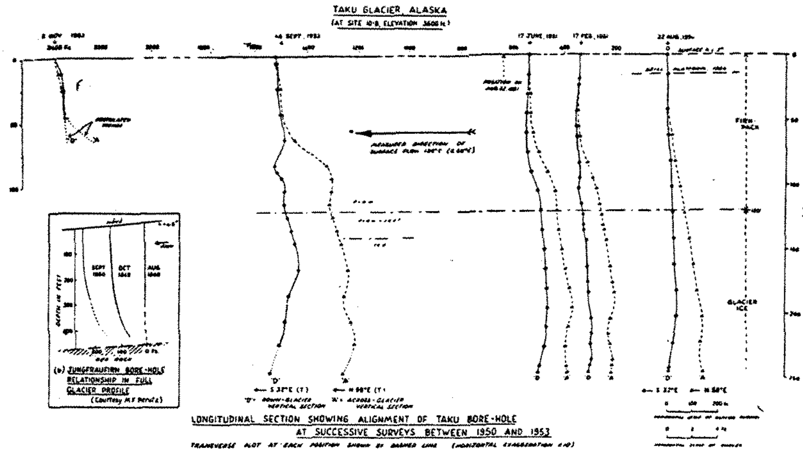


Fig. 4

TABLE I

Depth and Inclination Data on Taku Glacier Bore-hole Surveys

Date and Year	True Vertical Depth of Survey (2) (in ft.)	Time Increment Since Previous Record (months)	Max. Drift Angle to S.W.	Survey Spacing; Remarks
22 Aug., 1950	245	—	1°10′ (at 102 ft.)	20 ft.
11-17 Feb., 1951	244.96	6	2°00′ (at 94 ft.)	5 to 20 ft.
17 June, 1951	239.95	4	3°00′ (at 89 ft.)	15 ft.; protective casing lost in bottom 5 ft. of pipe.
14 Sept., 1952	222.73	15	7°15′ (at 84 ft.)	10 to 20 ft.; only false bottom attained 22 ft. from true bottom of pipe.
2 Nov., 1953	25	13.5	?	12 ft.; constriction in pipe at 25 ft. precluded deeper survey.

(2) Reference plane, 15 feet above wooden drill platform (Fig. 4).

All readings are within the manufacturer's designated accuracy of the inclinometer and have been plotted as true deformation (Fig. 4). This is on the assumption of no differential horizontal or vertical movement between the tubing and its encompassing firn or ice. Also duing the three years of accumulation involved, the change in vertical position of the upper end of the pipe with respect to the regime surface was small. The survey depths in this figure are therefore based on the initial reference level as if it were the true glacier surface.

At the time the pipe was installed, the maximum drift angle was 1°05′. When taking into account the down-valley glacier gradient (1° at Site 10B), the bore-hole was essentially perpendicular to the ice surface. This implies that all subsequent deformation of that portion of the pipe resting in solid ice cannot have been significantly affected by the initial alignment.

The Vertical Velocity Distribution

The deformation of the tube is considered to reflect the differential down-glacier movement of this ice mass at depth. A certain amount of vertical strain in the firn (compaction) may have accentuated the normal creep strain in the upper third of the tube, but this effect is considered slight.

Between initial and final surveys, the top of the pipe travelled a map distance of 2250 feet down-glacier in the direction 130°T. The actual course was somewhat irregular, with a total surface movement of 2378 feet. The mean aunual rate was 761 feet with a fairly uniform surface velocity prevailing throughout each year. The net rate of movement, however, was 10 per cent *greater* in winter, presumably due to the increased pressure of new accumulation.

Over short periods, faster rates and changes in direction of surface movement occurred. This is illustrated by the 1952 survey which shows that between July 19th and August 26th the flow direction shifted nearly 40° to a more southerly course. According to the triangulation, which is believed to be correct, the surface velocity increased at the same time to 4.2 feet per day. The cause of this is not clear, but it suggests relationship to an increased measure of extending flow either due to a change of load or to the fact that the glacier passed over a different configuration of the sub-glacial floor. (3)

The increments of relative movement between successive surveys at depth are illustrated in Figure 4. It is clear from this diagram that a somewhat greater flow rate occurs in the surface zone. In general, the vertical velocity distribution is similar to that measured in the upper half of the Jungfraufirn pipe in 1948-50 by Gerrard, Perutz and Roch (1952; v. inset in Fig. 4). Since there is a 1 to 10 horizontal exaggeration in the Taku Glacier curves, it may be seen that the differential velocities are not large. The inclination measurements, however, are more detailed and indicate certain features not found in the Jungfrau profiles.

Although the surface surveys show that the primary direction of mass movement is S.E. (down-glacier), the englacial surveys indicate that there is a marked lateral component of differential adjustment internally, in a cross-glacier direction towards the N.E. This is expressed as a gradual leaning of the pipe, reflecting a tendency of the ice to be diverted into a broad marginal basin (Icy Basin, Fig. 2). Half a mile east

(3) That abnormal «currents» or surges of flow may occur in portions of the glacier is suggested by the 1949 and 1950 velocity curves of Profile IV (Fig. 3). Note the narrow zone of faster moving ice, 2000 to 4000 feet west of Station 10B, a condition apparently not persisting into 1952. As discussed later, it is of interest to consider whether such variations in flow rate bear any relationship to varying glaciothermal conditions.

of the bore-hole, the névé inclines abruptly into this depression so that the sub-glacial floor is also assumed to slope away from the longitudinal trend of the valley. Although at Station 10B the névé is relatively flat, half a mile to the west the gradient inclines in the opposite direction towards the center of the trunk valley. Thus the pipe would appear to have passed along the crest of a low threshold delimiting the western edge of the tributary basin.

In order to distinguish the most significant trend of internal flow, the interplay of stresses resulting from the bedrock slope and configuration is briefly considered. For this purpose the tangent of the pipe's tilt angle has been plotted as a function of depth (Fig. 5). Only those values with drift directions lying within 20° of the

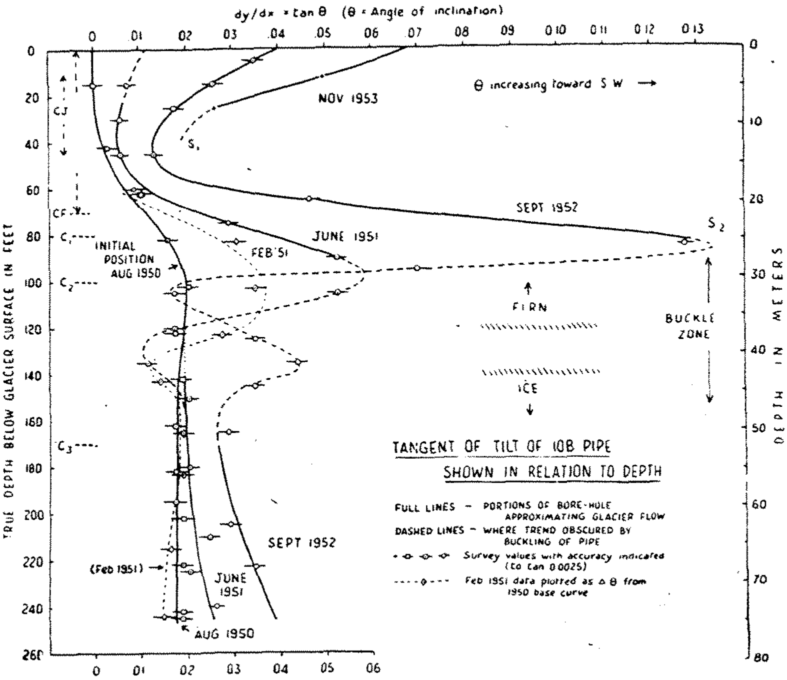


Fig. 5

cardinal directions N.E. or S.W. are used. Thus the figure represents the *maximum deformation* from the surface slope involved, and with the changes of inclination taking place in essentially one plane (the tilt angle in succeeding surveys increasing towards the southwest). The accuracy of the surveys is indicated to tan 0.0025, which is equivalent to the maximum percentage error in the inclination surveys. The results show that in this plane the *lowest 100 feet of pipe exhibit a much more uniform tilt than the upper 150 feet*, a fact of considerable importance to the interpretations.

Temperature vs. Deformation Anomalies

The most abrupt change appears as a flexure at 85 feet. It was at this depth that firn cores began to change over to those of firn-ice and bubbly ice. It is concluded that this flexure is largely the result of a relatively faster creep of firn over the deeper glacial ice, a condition not unlike that observed in seasonal snow-packs on bedrock (Haefeli, 1948). The flow pattern at shallow depth is complex, since a faster relative movement at the surface gradually decreases to a minimum at 45 feet and then rapidly increases again to a maximum at the 85-foot flexure. Such a reversal in flow gradient may be explained by the glaciothermal conditions indicated in Figure 6. Here the base of the annual cold wave in 1950-51 is shown to lie at 65 feet, which is equivalent to the CF level noted on the ordinate of Figure 5. The marked increase in deformation at and below this depth would thus appear to be a correlate of the temperature condition — i.e. with more rapid flow in the underlying or isothermal part of the firn-pack. Glen (1955, pp. 528-9) has shown experimentally the great sensitvity of the creep rate to temperature. For example, according to his painstaking laboratory tests the minimum rate of strain ($\dot{\gamma}$) of ice under constant stress (i.e. 6 kg cm^{-2}) varies with the temperature (t) in the ratio:

t	$\dot{\gamma}$ as a proportion of the value at 0°C.
at — 1.5°C.	13.5%
at — 6.7°C.	4.7%
at — 13.0°C.	1.0%

At an ice temperature of only 1°C. below freezing, the flow rate may be expected to be slowed down to approximately 1/5th that of conditions at 0°C. And at the maximum chill, shown in Figure 6 (—9.5°C.), the internal creep within the zone affected by the annual temperature wave should be less than 1/25th of that under the same stress at 0°C. With these proportions pertaining under conditions of unchanging stress they should be considered only qualitatively. For instance since the heat of deformation cannot escape from the interior of a temperate glacier, it may be generally assumed that there is local melting with a consequent weakening of crystal structure. It follows that the flow behaviour of an isothermal glacier will depend somewhat on its flow history, which in turn means that one field case may not be directly comparable with another. On the other hand, in an experimental thermodynamic treatment of the temperate state of ice, Steinemann (1957, personal communication) has found no reason to refute the simple extrapolation to 0°C. of flow data obtained under subfreezing conditions.

Thus it may be expected that *vagaries of the 10B tilt curve bear a strong relationship to the thermal conditions.* A further review of the pertinent field data appears to bear this out. For instance, not only does the depth of maximum annual penetration of the 1951 cold wave correspond with a pronounced flexure in the pipe, but also the slight temperature depressions noted at 80, 100 and 170 feet in Figure 6 lie at the C_1, C_2 and C_3 flexure points in Figure 5. This suggests that entrapped sub-freezing temperatures may in fact play a part in the differential flow of firn-ice (and ice) at depth. (4) Further to this the zone of delayed chill, which was retained up until the end of spring between the 15- and 50-foot depths (Fig. 6), corresponds in position with the uppermost flexure in Figure 5 and hence lies at the horizon of least velocity change (v. line CJ). *The noteworthy correspondence of these thermal anomalies with the most striking anomalies of deformation suggests not only a direct causal relationship but a similar temperature regime in each of the referenced years.*

Additional flexures occur in the tangent of the tilt plot at the 135-foot and 225-foot depths. Original alignment produced in the drilling may explain the lowermost

(4) The correspondence of position argues against the possibility of these anomalies being due to thermistor error.

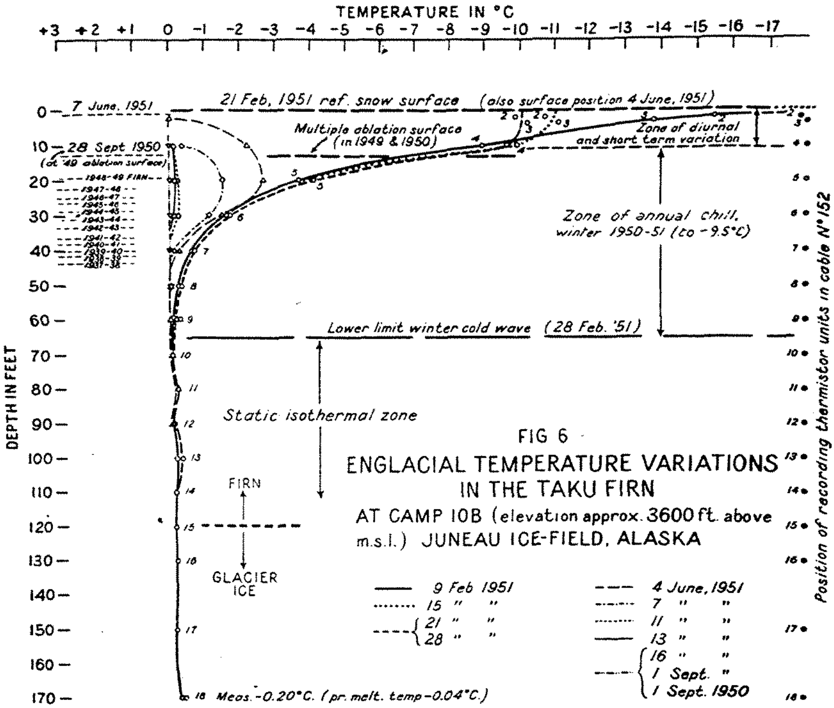


Fig. 6

anomaly, but that at 135 feet is more likely due to excessive bending of the pipe at the transition point between the firn-pack and deeper solid ice. (Hence referred to as the «buckle zone» in Fig. 5). The possibility is minimized, therefore, that either of these flexures bears any genetic relationship to a discontinuity in the thermal profile.

It is of further interest that the dominant direction of surface movement given by the theodolite surveys is 130°T., while the englacial records show the maximum deformation in the direction 150°T. Most of the deformation may have occurred during those shorter periods when the pipe was transported along the more southerly course. However, it would seem more likely that a torsional stress has been set up as a result of the easterly component of firn creep acting on the top of the pipe in one direction while the southerly component of ice flow at depth affected it in another. A further possibility is that this effect is due to a vertical shear strain superimposed on the flow strain as a result of some settling in the firn. The resulting differential movement may have served to twist the top of the pipe in a counter-clockwise manner. Regardless of the actual nature of the stressing conditions, there is a marked difference in the recorded deformation of the lower half of the pipe *imbedded in solid ice compared to that in the upper half imbedded in firn. Therefore, only the data from the lower section are used as a key to the stress/strain conditions in the glacier as a whole.*

Creep Rate as a Function of Load

The data in Figure 6 show that the ice at depth in the 10B profile is essentially temperate — i.e. isothermal at the pressure melting point. The fundamental relationship as far as the creep rate is concerned is, therefore, the dependence of rate of strain on the load. In the analysis the first step is to resolve the curves of Figure 5 into a figure of strain as a function of the stress. This is done on the assumption that ice behaves as a plastic solid and with the starting consideration that the strain rate may be calculated as a problem in pure shear (Nye, 1951). An admirable study of the creep of polycrystalline ice has recently been made on laboratory samples by Glen (1955) who has shown that the rate of strain is basically the result of the thermal motion of molecules tending to re-arrange themselves in a pattern conditioned by the stress. That these laboratory tests serve at least as a working basis for the analysis of glacier flow has been reasonably demonstrated by the comparison of theoretical results with those achieved in field measurements from bore-holes and tunnels in Alpine and Norwegian glaciers (Nye, 1953). Accordingly, as was first suggested by Perutz (1950), the exponential relationship between the stress and the deformation per unit time is taken as:

$$\frac{d\gamma}{dt} = k\ \tau^n$$

where γ is the shear strain, τ is the shear stress in bars, k is a constant for any given temperature and n is an empirical constant depending in large measure on the physical character of the ice (i.e. its texture, intergrowth complex, grain size, and preferred crystal structure). Also, the exponent n probably depends to some degree on the magnitude of the stress, a factor considered by some to be significant only in very deep or otherwise highly stressed ice. (5) In this case $\frac{d\gamma}{dt}$ is considered as the *strain per year*, and is designated as $\dot{\gamma}$. The function is readily calculated as radians of angular deformation per year, since it is numerically equivalent to the tangent of the inclination angle θ in Figure 5.

Although from Steinemann's experiments (1954, p. 449) it must be expected that the power law used with a universal constant n can give no more than a good approximation of the true strain rate at depth (also v. previous footnote), it is probable that because of the simple form and known thicknesses involved in the Taku Glacier fairly representative results can be obtained through a measured mean value of the factor n. By applying the formula, therefore, it is considered that a reasonable determination is made of the movement *within the glacier*. This computation should at

(5) It has been mentioned that Glen's experiments have demonstrated the rapid increase in flow near 0° C. This is because within the temperature limits 0.02° to —13.0° C. k is largely a function of temperature. For the temperature range from —1.9° to —22° C. and larger stress limits, Steinemann (1956, and personal communication) verified Glen's general conclusions through compression and tension tests on polycrystalline natural ice (melted snow). As Glen has done, he distinguishes a secondary or quasi-viscous flow and a tertiary or paracrystalline (metamorphic) flow. During the latter phase at varying stresses, he observed the structural behaviour of his samples to be unstable. He proposes that the tertiary flow is identical to processes in a glacier—apart from a supplemental stressing which would affect k and probably also the exponent n. *From his considerations, for both stages the flow law would have an exponent n which, depending on the stress, varies from 1.6 to 4 for stresses from 0.7 to 17 kg cm*$^{-2}$. Additionally, Steinemann has evidence that the plasticity of single ice crystals is influenced by the type and magnitude of impurity in the water from which the ice has crystallized (a decrease in purity causing an increase in k). This effect, however, must be vanishingly small in the present field case, since the only significant impurities in firn and ice samples of the 10B bore-hole were chloride and NaCl in the minute proportions of 0.3 to 7.4 p.p.m. (Miller, 1953, pp. 33-36).

In Glen's laboratory experiments, at —0.02° C. and with a load of 1 to 10 kg cm^{-2}, the constants were: $n = 4$ and $k = 0.25$. The values of each constant as used in the Taku Glacier analysis are discussed in the following pages. It will be shown that the value of n in this particular case approximates the mean of the range found by Steinemann.

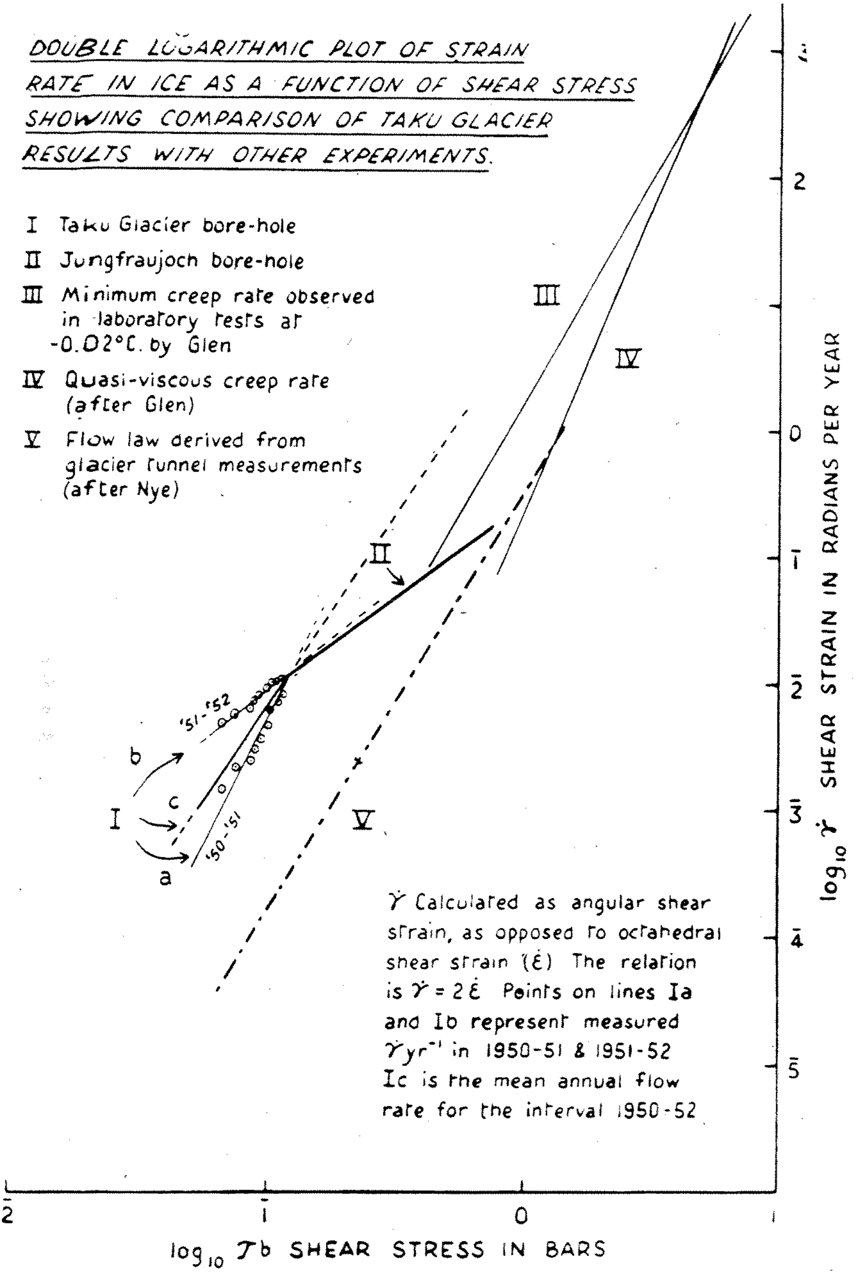

DOUBLE LOGARITHMIC PLOT OF STRAIN RATE IN ICE AS A FUNCTION OF SHEAR STRESS SHOWING COMPARISON OF TAKU GLACIER RESULTS WITH OTHER EXPERIMENTS.
I Taku Glacier bore-hole
II Jungfraujoch bore-hole
III Minimum creep rate observed in laboratory tests at -0.02°C. by Glen
IV Quasi-viscous creep rate (after Glen)
V Flow law derived from glacier tunnel measurements (after Nye)
γ̇ Calculated as angular shear strain, as opposed to octahedral shear strain (ε̇) The relation is γ̇ = 2ε̇ Points on lines Ia and Ib represent measured γ̇yr⁻¹ in 1950-51 & 1951-52
Ic is the mean annual flow rate for the interval 1950-52
'51-'52
'50-'51
a
b
c
I
II
III
IV
V
log₁₀ γ̇ SHEAR STRAIN IN RADIANS PER YEAR
log₁₀ τb SHEAR STRESS IN BARS
2
0
1̄
2̄
3̄
4̄
5̄
0
1

Fig. 7

least be more reliable than for glaciers of complex morphology. Also, by combining the englacial movement data with the surface velocity and geophysically determined depth records, an assessment is made of the relative proportion of movement through *slippage on the bed.*

The shear stresses indicated in the 1950-51 and 1951-52 regime years have been derived from the curves of Figure 5, assuming from the depth profiles that the glacier lies on a gently sloping bed without significant relief. The *shear stress* at each inclinometer level is calculated from the basic formula in dynes cm^{-2}:

$$\tau = D\rho g \sin \alpha$$

Where D is the depth in centimeters, ρ the bulk density of the overlying mass (6) α the surface gradient and g the gravitational constant.

The results are shown in Figure 7 (Plot I) as a double logarithmic plot of the strain rate against shear stress. Two sets of curves (I*a* and I*b*) were obtained by taking the difference between the smoothed full-line curves of 1950-51 and 1951-52 in Figure 5, thus giving a comparison between the englacial flow rates for the first two years of record. Since the concern of this paper is the strain relationship in solid ice, the plottings represent only the depths between 120 and 245 feet. (7) The values obtained from the actual inclinometer readings are indicated by small circles in the figure to show the close agreement with the smoothed plot. It is of interest that the points in each year's record are all on a straight line and that the shear stresses involved are all below 1 bar. (8) As was found by Gerrard, Perutz and Roch in the Jungfraujoch bore-hole (1952, p. 554), there is no indication of a sudden yield stress and the slope of each line is much steeper than would be the case were we dealing with a Newtonian liquid. In fact, by comparing the 1951-52 curve (line I*b*) with the slope calculated by Perutz for the Jungfraujoch bore-hole (line II) we find very close agreement. This is of interest because the results both apply to temperate ice.

It is of further interest to compare the Taku Glacier results with the compression tests carried out by Glen, and with the flow law derived by Nye (1953) from analysis of glacier tunnel measurements. The comparison is made in Figure 7, which shows a double logarithmic plot of the data for each of the other experiments, presented as a log of the strain rate against the log of the shear stress. To facilitate the comparison a plotting of the mean flow rate per year in the Taku Glacier example is also given for the interval 1950 to 1952 (line I*c*). This may be considered as a measured mean creep rate at the bore site because of the uniform increase in surface gradient between these two years. In this figure, $\dot{\gamma}$ is shown as the angular shear strain rate as opposed to octahedral shear strain rate $\dot{\varepsilon}$. (9) The relation is $\dot{\gamma} = 2\dot{\varepsilon}$. The minimum creep rate observed in Glen's laboratory tests at —0.02°C. is shown (line III) in comparison with the quasi-viscous creep rate (line IV, also from Glen's experiments) and the flow law relationship after Nye (line V). The mean annual strain rate from the Taku measurements is closely parallel with that of line V, although representing a considerably lower range of stress.

In each of these lines, a different value pertains for the empirical constants n and k. From the cited laboratory test, the quasi-viscous line may be considered as the idealized relation in which the full stressing conditions are known. As such it provides us with an established reference, any deviations from which may be attributed to a difference in the law, or to the presence of extending and compressive

(6) From graphic integration of field data and extrapolated density curves in the vicinity of the bore-hole.

(7) An analysis of the strain relationship in the overlying firn-pack has been presented elsewhere (Miller, 1956, ch. IX, c-5).

(8) i. e. 10^6 dynes $cm^{-2} \simeq 1$ kg $cm^{-2} \simeq 1$ atmosphere.

(9) Sometimes referred to as the «effective strain rate» (v. Nye, 1953, pp. 478-9).

forces (Nye, 1952) which have not been taken into account. It is not to be expected that line I is analogous since, as one proceeds down-glacier from site 10B, there are an increasing number of tension crevasses across the line of flow. Additionally, there is an increase of 0.4° in gradient at the 1953 bore site compared to 1950. These facts, and the apparent twisting of the pipe as previously discussed, imply that the bore-hole was moving from an area of some extension to greater extension. In the Jungfraufirn, the rate of extension was measured as 14 per cent per year. The close agreement between lines *Ib* and II suggests that a similar rate of extension prevails in this sector of the Taku, where no such measurement has as yet been made. From these considerations it would appear that the pure shear analysis rather *over estimates* the relative importance of englacial shear stress. A comparison between theoretical and observed values is therefore useful for giving an estimate of the effects of extending and compressive flow.

For the basic comparison, n and k are derived from line IV and from lines I*a*, I*b* and I*c* in Figure 7. The value of n represents the slope of each line. The constant k is obtained from the fundamental stress/strain formula at the stress of 1 bar. Since the logarithm of 1 is zero, k is the ordinate at the point where the abscissa of any log stress/log strain rate line is zero. The constants in each of these cases and in the relationship given by line V are:

	n	k
Line I*a* (10B, 1950-51)	4	56.2
Line I*b* (10B, 1951-52)	1.6	0.4
Line I*c* (10B, *mean*, 1950-52	3	7
Line IV (Quasi-viscous creep rate, from laboratory tests)	4	0.25
Line V (Flow law, derived from tunnel measurements)	3	0.33

Although the mean value of n at site 10B is somewhat lower than the experimental quasi-viscous rate in compressive flow, it is in agreement with the exponent of effective shear stress found by Nye to pertain in the tunnel tests. This supports that the quantity $n = 3$ is of an order close to the average field condition usually found in low gradient temperate glaciers. Therefore, by assuming this value as reasonable, and by extrapolating the relationship of line I*c* all the way to the base of the glacier, a determination is made of the total flow *within the glacier*. (It is reiterated that this is on the ad hoc assumption that the simple power law for general analyses may be considered to govern the englacial deformation rate at depth.) The same is done for each set of data. For this calculation the difference is found between the *plastic flow velocity at the measured seismic depth* (U_D) and the *surface velocity* (U_o) at the top of the bore-hole. The relation is expressed by combining the empirical constants with the stress at this depth(D) in the manner suggested by Nye (1952, p. 86):

$$U_o - U_D = \frac{k}{n+1} D \, \tau^n$$

The Proportion of Basal Sliding

As far as the total transfer of ice across this profile is concerned, the differential flow within the glacier is seen to be small between the base of the pipe and bedrock. This is well demonstrated in part II of Table II, and is a fact directly related to the low gradient in this sector. The effect of increased slope is illustrated by the figures based on the quasi-viscous creep law, showing an increase of 1 to 36 per cent of englacial movement with a change of surface gradient of only 1° to 3°. Thus, in theory, there is verification of the small mass transfer accredited to differential movement on the pipe profile — i.e. in the upper quarter of the glacier. In contrast, there is the substantial surface velocity which must be explained. Since the calculations in Table II are based on what is considered a reliable determination of depth, the *dominant translation of material is concluded to be from «sliding» of the glacier on its bed.* [10] On the Aletsch Glacier, near the Jungfrau bore-hole, the average gradient was 4.6°, with basal slippage calculated to account for about one-half of the surface velocity. On the Taku Glacier, which is similar in form, but with an average gradient of considerably less, a greater proportion of mass movement is by sliding. According to the mean calculated from the measured 1950-52 deformation (v. Part I of the table),

TABLE II

The Amount of Movement Within the Glacier According to The Measured and Theoretical Rates of Shear Strain

	Movement Within the Glacier		Shear Stress on Bed in bars	Basal Sliding, as Proportion of Total Movement
	(in meters)	(in % of total)		
I. According to extrapolated creep rate from pipe data				
1950-51 (line I*a*)	166	76	0.51	16%
1951-52 (line I*b*)	22	5	0.51	92%
Est. mean of I*a* I*b*	—	—	—	54%
Calculated mean over 1950-52 (line I*c*)	76.5	23	0.51	*67%*
II. According to quasi-viscous rate from laboratory tests				
assuming 1° slope, as on seismic profile	1	1—	0.51	*99%*
assuming 2° slope	17	7	1.0	93%
assuming 2.5° slope	34	15	1.2	85%
assuming 3° slope	84	36	1.5	67%
III. According to rate found in tunnel measurements				
assuming 1° slope, as on seismic profile	4	1	0.51	*98%*
assuming 2.5° slope	47	20	1.2	80%

[10] The term «sliding», as hereafter used, is applied loosely since in some glaciers the movement can in large measure be due to intensified differential shearing in a thin zone of tectonic foliation close to the bed. The writer has observed such in a section one meter thick at the rock contact in the tunnel of Skautbreen, Norway (v. McCall, 1952, Fig. 5), and in sections up to 40 feet thick on certain Greenland glaciers. Such cases may be accentuated by a higher value of the k constant in the creep law, as a result of impurities in the water of the ice at the sole of the glacier (v. footnote page 446). The effect is the same, however, as if yielding were by pure slip at the bedrock interface.

two-thirds of the down-glacier movement at Station 10B is due to bottom slip, or to basal shearing. This leaves one-third consequent on flowage within the main body of the glacier. These ratios are, of course, based on the mean surface slope of 1°. At the same gradient, the theoretical values are compared, including those given by Nye's flow law (v. Parts II and III of Table II). On this hypothetical basis, 98 to 99 per cent of the mass movement would be due to bottom slip and even by using the average overall gradient of the main branch of the Taku Glacier (2.5°), 80 to 95 per cent would still be accorded to basal slip. On empirical grounds, such high ratios cannot be justified on Movement Profile IV (Fig. 3). Therefore. *the mean percentage of basal sliding calculated from the field measurements is considered to pertain. It is of interest that this value is 32 per cent less than that determined from the quasi-viscous rate.*

A Trajectory of Longitudinal Stress

The actual extent to which bottom sliding takes place depends not only on the surface gradient but on the slope and roughness of the underlying topography, on the width and thickness of the ice in question, and on any blocking factors involved, such as a tributary glacier abutting against a trunk glacier at the junction of two valleys. All these elements, of course, vary in different sectors of the icefield. Thus the amount of slippage on the glacier floor at the bore-hole is not to be construed as representative, even though for purposes of analysis this profile has been our essential concern. To illustrate this point a comparison is made with the transects on the main lower glacier. On Profile II an even higher ratio of sliding is calculated from the empirical law — i.e. 92 per cent. On another profile, two miles above the terminus, 54 per cent is indicated. The difference in these ratios may be explained by the narrower channel and increased depth of ice near sea-level.

Substantial slippage and marginal shearing also take place along the edges of the glacier. This is suggested by the form of the transverse surface velocity curves in Figure 3. Additionally in support of the effectiveness of true bottom slippage are the following facts: (1) the deeply entrenched and U-shaped channel of the lower glacier (Fig. 1, especially the sector down-glacier from Site 11*a*); (2) a noteworthy display of deep grooving and lateral striations along the bedrock margins; and (3) the vast quantity of rock flour which is constantly silting into Taku Inlet along the lines of the glacier's advancing front and which produces a characteristic milky color in the water of the fiord for some miles out. *None of these conditions could develop without considerable abrasion from the glacier's actually sliding on its bed.*

As for the basic cause of the high ratios of slippage described, it is clear that *flow within the glacier at a slope of 1° is so small that other stresses than those explained by pure shear must be in effect. Any important influence by hydrostatic pressure is ruled out, on the basis that it is as negligible in ice as in liquids.* (11) *The most significant supplemental stress can probably be attributed to a strong longitudinal force superimposed on the normal laminar flow and produced by an excessive increase in accumulation on the higher névés of the Taku-Llewellyn Glacier system in the past 100 years. To this stress tensor, and probably to an attendent increase in temperature within the ice itself, the phenomenal 20th-century advance of the Taku Glacier has been ascribed* (Miller, 1954*b*).

(11) A conclusion cited as probable by Nye (1952, p. 82) and recently verified by Steineman (1956, personal communication) in laboratory tests at the Weissfluhjoch Research Station. Steinemann's experiments were made in a torsional-shear apparatus at a temperature of —1.9° C. and have shown that there is no influence whatsoever from *confining pressure* (up to 90 atmospheres) either on the actual flow rate in ice or on the accelerating or decelerating phases of recrystallization.

ACKNOWLEDGEMENTS

The writer is much indebted to Dr. John Glen of Cambridge University and Dr. Samuel Steinemann of Laboratoire Suisse de Recherches Horlogeres, Neûchatel, for helpful discussions and criticism in connection with this research and for their particulai kindness in making available the results of unpublished laboratory tests. Special appreciation is also extended to Dr. Henri Bader for his advice and personal support in connection with the drilling program. Acknowledgement is likewise given to the following peisons whose untiring cooperation or assistance in the field greatly facilitated this research: Dr. Donald Davidson and Mr. Anders Anderson of the Longyear Co.; and Dr. Gerald Wasserburg, F.A. Milan, F.A. Small, the late Dr. Arthur Gilkey, and other members of the 1950-53 expeditions to the Juneau Icefield.

The bore-hole investigations were made possible by a field grant fiom the Geological Society of America, and by contract funds from the U.S. Office of Naval Research. The American Geographical Society, the U.S. Geological Survey, the Eastman Oil Well Survey Co., and Columbia University provided materiel aid. The Foundation for Glacier Research, Inc., made available the necessary funds for preparation of this paper and related reports.

REFERENCES

GERRARD, J. A. F., PERUTZ, M. F. and ROCH, A. (1952) Measurement of the Velocity Distribution along a Vertical Line through a Glacier, *Proceedings of the Royal Society*, A, Vol. 2/3, pp. 546-558.

GLEN, J. W. (1955) The Creep of Polycrystalline Ice, *Proceedings of the Royal Society*, A, Vol. 228, pp. 519-538.

HAEFELI, R. (1948) The Development of Snow and Glacier Research in Switzerland, *Journal of Glaciology*, Vol. 1, No. 4, pp. 192-201.

— (1952) Observations on the Quasi-Viscous Behaviour of Ice in a Tunnel in the Z'Mutt Glacier, *Journal of Glaciology*, Vol. 2, No. 12, pp. 94-99.

MCCALL, J. G. (1952) The Internal Structure of a Cirque Glacier, *Journal of Glaciology*, Vol. 2, No. 12.

MILLER, M. M. (1951) Logistical Operations, Juneau Ice Field Research Project, Alaska, 1949, 1950 and 1951, *J. I. R. P. Report No.* 5, American Geographical Society, New York.

— (1953) Juneau Ice Field Research Project, 1951 Winter Season, *J. I. R. P. Report No.* 8, American Geographical Society, New York.

— (1954*a*) Mechanical Core Drilling in Firn and Ice (with a report on related investigations in the Taku Glacier, S. E. Alaska, 1950-53). Mimeographed report prepared for the E. J. Longyear Co., The Eastman Oil Well Survey Co., and the Geological Society of America, 27 pp.

— (1954*b*) Glaciothermal Studies on the Taku Glacier, Southeastern Alaska, Publication No. 39 de l'Association Internationale d'Hydrologie, *U. G. G. I.*, Tome IV, pp. 309-327.

— (1956) The Glaciology of the Juneau Icefield, Southeastern Alaska, unpublished Ph. D. dissertation, The University of Cambridge, 2 Vols., 750 pp.

NYE, J. F. (1951) The Flow of Glaciers and Ice-sheets as a Problem in Plasticity, *Proceedings of the Royal Society*, A, Vol. 207, pp. 554-572.

— (1952) The Mechanics of Glacier Flow, *Journal of Glaciology*, Vol. 2, No. 12, November, pp. 82-93.

— (1953) The Flow Law of Ice from Measurements in Glacier Tunnels, Laboratory Experiments and the Jungfraufirn Borehole Experiment, *Proceedings of the Royal Society*, A, Vol. 219, pp. 477-489.

PERUTZ, M. F. (1940) Mechanism of Glacier Flow, *Proceedings of the Physical Society*, Vol. 52, pp. 132-135.

— (1950) in Symposium on Glaciology—the Flow of Glaciers, *The Observatory*, Vol. 70, No. 855, pp. 64-65.

STEINEMANN, S. (1954) Flow and Recrystallisation of Ice, Publication No. 39 de l'Association Internationale d'Hydrologie, *U. G. G. I.*, Tome IV, pp. 449-462.

— (1956) Dissertation on the Plasticity of Ice, in press (v. Steinemann, Laboratoire Suisse de Recherches Horlogeres, Neûchatel).