A Comparison of Methods for Determining the Surface Mass Balance of a GPS Surveyed Movement Profile

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Abstract

Two methods are described that allow the mass balance of a GPS surveyed movement profile to be determined. One method relies on the interpolation of estimated surfaces derived from survey measurements, while the other method derives mass balance via summation of the volumes of trihedra. During the 1993 Juneau Icefield Research Program field season a transverse profile on the Taku Glacier was surveyed via GPS and theodolite/EDM techniques over a period of five days. Easting, northing, and height coordinates were collected at 27 points along 2 parallel lines of movement stakes. The net loss of firn and mean daily ablation during the survey period was calculated using the two methods. Net loss calculated via the interpolation method was 257,234 m^3 (~128,617 m^3 water equivalent) while the trihedral method gave a net loss of 226,980 m^3 (~113,490 m^3 water equivalent). The advantages and disadvantages of each method are discussed, and the interpolation method is found to give a better approximation of the true mass balance state.

Introduction

Investigations of mass balance are one of the most important factors in understanding the behavior of glacial systems in response to short-term and long-term climate trends. Because the overall health of a glacier system is directly related to the mass balance of the glacier, methods to accurately calculate the mass balance are necessary.

The method most often used on the Juneau Icefield is to determine the net accumulation remaining at the end of the ablation season (Pelto and Miller, 1990). This is done by digging numerous test pits across the icefield to determine the depth of firn to the previous year's surface. At each test pit samples of firn are collected at 5-10 cm intervals and the volume, mass, and density are determined. The water equivalent is then determined for each sample and the overall water equivalent of firn above the previous year's surface is calculated. Integration of the water equivalent at each test pit site gives the net accumulation remaining at the end of the ablation season.

This paper discusses two alternate methods whereby position and elevation data obtained from global positioning system (GPS) surveys can be used to determine the mass balance of a transect across a glacier. One method relies on interpolation to construct a model of the glacier surface at various points in time. The volume of the survey profile above an arbitrary horizontal reference plane is computed for each survey epoch, and subsequent comparison of the volume of each epoch gives the net change in volume. The second method, which is referred to in this paper as the trihedral method, does not rely on interpolation to construct the surface model. Rather, the easting, northing, and elevation coordinates for each flag are used to construct a series of surface triangles. The surface is thus defined only by the actual flag coordinates—intermediate points are not considered and interpolation is not performed. As with the interpolation method, the volume above an arbitrary horizontal reference plane is calculated and the net change is determined by comparing the volumes of the individual survey epochs. Each method is discussed in detail in the following sections.

Study Area

The Taku Glacier originates in the Juneau Icefield, which trends north 120 km from Juneau, Alaska. The icefield extends from 58° 20´ to 59° 30´ north latitude, covering an area of approximately 4,000 square kilometers (Figure 1). Some 38 main glaciers radiate from the central névés at an elevation of 1,800-2,200 meters, several of which, including the Taku Glacier, terminate near tidewater along the southern boundary of the icefield.

The Taku Glacier is the largest glacier on the icefield with a length of some 50 km and covering an area of approximately 671 km^2. It extends from the crestal névé at an elevation of 1,800 meters to tidewater along the southern extent of the icefield. The surveyed profile was located along Profile IV, an established transect at an elevation of approximately 1,100 meters. This profile was selected due to its close proximity to Camp 10, and because it has been the focus of intense study from 1946 to the present. Seismic, gravimetric, and ice radar surveys, borehole studies, and movement and strain rate surveys have been conducted along this transect.

Survey Methods

The first step in performing the survey was the establishment of the survey profile. Twenty-seven movement stakes, in two roughly parallel lines, were placed perpendicular to the glacier flow. The lines were offset so as to form a series of triangles between the two lines of stakes, thus providing the data needed to calculate movement, mass balance, and strain rates. Figure 2 shows the profile configuration and its location with respect to the surrounding geography.

The stakes were then surveyed using both GPS and theodolite/EDM methods. The GPS surveys utilized differential methods and were carried out in “rapid static” mode using Leica 300 GPS receivers. A reference receiver was placed at control point No. 19.1 of the Taku local network and maintained a lock on a minimum of four satellites during the survey. A roving GPS receiver was placed at each of the 27 survey flags and collected data at 15 second intervals for

Figure 1: Map of the Juneau Icefield (Molenaar, 1990).

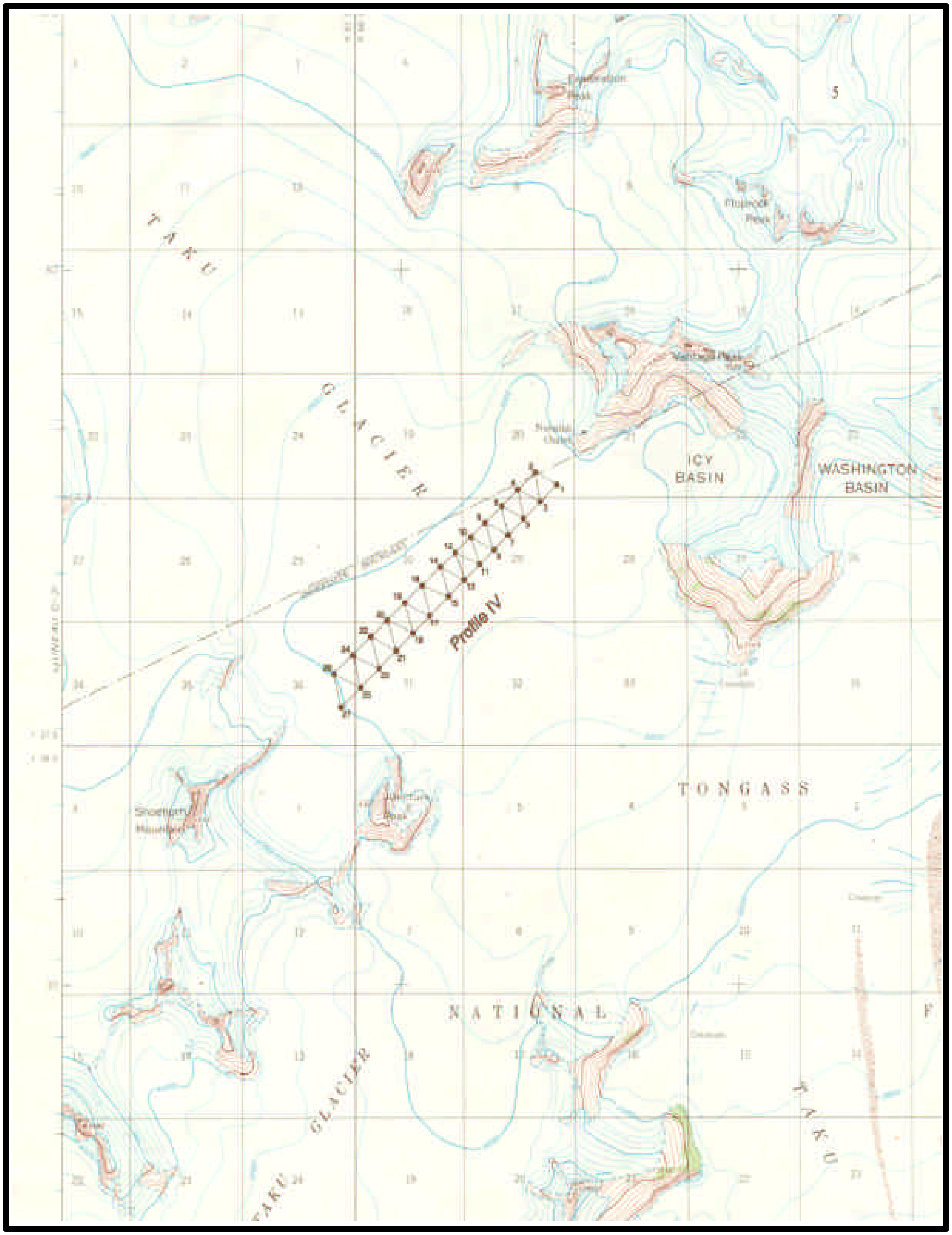

Figure 2: Geometry of Taku Profile IV, showing approximate flag positions. The Taku Glacier flows from the northwest to the southeast.

15 minutes, for a total of 60 measurement epochs per flag. Raw survey data from both the reference and roving receivers was then downloaded to a personal computer and post processing of the reference and roving GPS observations determined the baselines between the reference point and the flags, and the latitude, longitude, and height of the survey flags. Latitude and longitude coordinates were transformed to a coordinate system based on the JIRP projection, similar in nature to the Universal Transverse Mercator projection. Appendix 1 lists the GPS derived easting and northing coordinates, and the height of each flag for the initial survey and the resurvey.

Concurrently with the Epoch 0 GPS survey, a theodolite/EDM survey was also carried out. This was done as a safety factor—if problems developed with the GPS equipment we would still be able to obtain movement, strain, and mass balance data from the theodolite surveys. An additional benefit of performing the theodolite survey was that movement vectors derived from the GPS and theodolite surveys could be compared. Measurements consisted of the horizontal angle between the reference point (No. 19.1) and the individual survey flags, the zenith angle from the control point (No. 19) to the flags, and the slope distance from the control point to the flags. Reduction of the survey data followed the method outlined by McGee (1992) and easting, northing, and height coordinates were calculated for each flag during each survey epoch. These coordinates are presented in Appendix 2. A comparison of the movement vectors derived from the GPS surveys and the theodolite/EDM surveys is presented in Appendix 3. Results of the comparison show a greater overall movement from the GPS surveys than from the theodolite/EDM surveys. Additionally, the GPS derived movement vectors are not as erratic as that obtained by theodolite/EDM. This is because sighting errors are eliminated with the GPS method. Thus the GPS surveys provide a closer approximation of the true movement, particularly as the distance increases away from the survey station.

The Interpolation Method

Surface modeling is an excellent way in which to quantify the temporal and spatial variations of a glacier because it allows the visualization and measurement of the surface morphology in three dimensions. Modeling of the true surface is based upon the X, Y, and Z coordinates of surveyed points the greater the number of points, the closer the estimated surface will be to the true surface. Thus a large number of surveyed points, relative to the study area, will result in a surface that very nearly matches the true surface. In performing glacier surveys however, the number of points that can be surveyed is limited by time, weather, and logistical constraints. Because of this, interpolation is required to derive the estimated surface.

The surface for the area within Taku Profile IV was interpolated using the linear kriging algorithm contained in the *Surfer (version 5.0)* computer program. This is an advanced interpolation procedure which generates an estimated surface from a scattered set of surveyed points and their associated elevations. The surface is defined by a regularly spaced grid composed of 63,001 discrete grid points, with a cell size of 12 x 12 meters. The surface area, volume, and net loss of firn for Profile IV was determined as follows:

1) The first step was to construct two regularly spaced raw grids (one for each survey epoch) derived from the survey data. The GPS survey data used for the generation of the two grids is shown in Appendix 1. For each survey epoch the easting coordinate, northing coordinate, and height of each flag were used as the basis for interpolating the unknown portion of the grids. The interpolation parameters used were as follows:

Method:	Kriging (linear)
Grid size:	251 rows x 251 columns
Cell size:	12 meters x 12 meters
Easting range:	484,800 to 487,800
Northing range:	6,500,400 to 6,503,400
Search radius:	4,000 meters
# of nearest points:	27

   This resulted in two regularly spaced grids (one for each survey epoch), each covering an area of 9 km^2. Each grid point is defined in three dimensions by an easting coordinate, northing coordinate, and a height value.

2) The accuracy of the interpolation depends in part upon the density of the survey points in areas with many points, the interpolation is more accurate, while in those areas with fewer points the interpolated grid points are less accurate. Because of this it was necessary to disregard those grid points that were not within the area of the survey profile. This was done by using a "blanking file" to create a secondary grid with the same geographic extent as the survey profile. The blanking file is a space delimited ASCII file of easting and northing coordinate pairs. These coordinates define the boundary of a polygon which encloses the spatial extent of the survey profile and reduces the first grid created from 9 km^2 to 0.9 km^2. The blanking operation sets the Z value of all grid points outside the polygon to zero. Only grid points with non-zero Z values are used in the surface area and volume computations. The coordinates used in the blanking file are shown in Appendix 4. The resulting blanked grid is shown in Appendix 5.

3) The volume of the profile is defined by the easting and northing coordinates of the blanking file, the elevation of the estimated surface, and a base elevation of 1,095 meters. Using these boundaries, the volume for each survey epoch was computed using the volume computation algorithms in *Surfer*. These consist of the trapezoidal rule, Simpson's rule, and Simpson's $^3/_8$ rule. Utilizing all three methods allows an estimation of the accuracy of the volume computations to be made. Given the function $f=a-b$ and the standard deviations s_a and s_b, the standard deviation of $f\ (s_f)$ is found by $s_f=(s_a^2+s_b^2)^{0.5}$. The volume of firn lost DV is then easily calculated by subtracting the volume for Epoch 1 from the volume for Epoch 0. The mean volume of firn lost due to ablation during the survey period was 257,234 m^3, ± 11,146 m^3. The data are shown in Table 1.

Epoch	Trapezoidal Rule	Simpson's Rule	Simpson's $^3/_8$ Rule	Mean	St. Dev.
July 20, 1993	14,035,900	14,049,500	14,035,700	14,040,367	7,910
July 25, 1993	13,778,700	13,792,200	13,778,500	13,783,133	7,853
			DV (m^3)	257,234	11,146

Table 1: Volume of Profile IV for each survey epoch and net change in volume as calculated by the interpolation method.

4) The last step was to calculate the surface area of the movement profile. The surface area was calculated using *Surfer* and is that area of the surface enclosed by the blanking file coordinates. The *Surfer* calculation method includes variations in area due to slope. This provides a better estimation of the surface area than a simple rectilinear computation method because, for a given level area, the surface area will increase as the slope of the surface increases. Thus an area with a non-level surface will have a larger surface area than a similar area with a level surface. After the surface area of the profile is determined, the ablation rate can then be calculated. Given the change in volume of the movement profile DV, the original surface area A_p, and the elapsed time T in hours between survey epochs, the ablation rate per day A is determined by

$$A = 24\left(\frac{\frac{\Delta V}{A_p}}{T}\right)$$

and the resultant ablation rate is 5.33 cm per day. The final data for Profile IV as derived from the interpolation method are shown in Table 2.

DV (m^3)	St. Dev. (m^3)	Surface Area (m^2)	Ablation Rate (cm/day)
-257,234	11,146	965,400	5.33

Table 2: Change in mass balance of Profile IV from July 20, 1993 to July 25, 1994 as calculated via the interpolation method.

The Trihedral Method

While interpolation is an excellent tool for visualizing and quantifying a surface, one drawback is that the accuracy of the interpolated surface is dependent upon the accuracy and appropriateness of the interpolation algorithm and the various parameters used in the interpolation. Residuals introduced by the interpolation can significantly reduce the accuracy of the surface, thereby increasing the probability of drawing inaccurate conclusions concerning the profile's mass balance state.

Like interpolation, this method uses the easting, northing, and height coordinates of the flags to determine the change in volume. However, rather than using interpolation to approximate the surface in areas where survey data is missing, only the actual surveyed areal extent and height of the flags is used. A much simpler representation of the glacier surface is thus obtained whereby the surface is composed of a series of planar triangles, the points of which are defined by the X, Y, and Z coordinates of the flags. Figure 3 shows a hypothetical surface constructed with this method. Figure 4 shows a single trihedron and the various elements integral to the calculation of the volume and surface area.

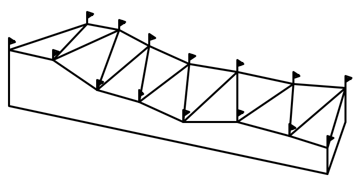

Figure 3: Movement profile composed of a series of individual trihedra.

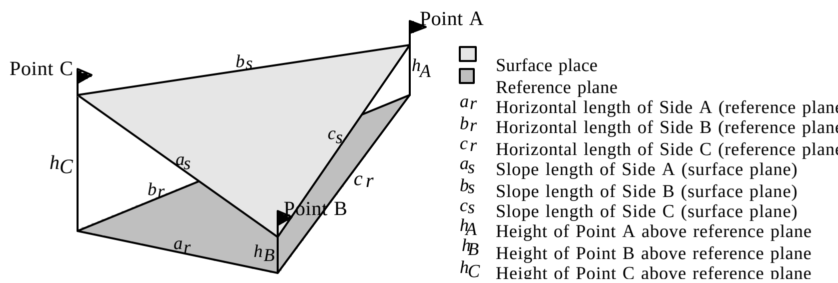

Figure 4: An individual trihedron. The surface area and volume of each trihedron is summed to derive the volume and surface area of the complete profile.

As can be seen, the complete profile is composed of several individual triangles, or trihedra. The areal extent of each trihedron is defined by the easting and northing coordinates of three survey flags, and the volume is defined by the elevation of the reference plane and the surveyed easting, northing, and height of the flags. Summation of the surface area and volume of each individual trihedron gives the surface area and volume above the reference plane of the entire profile. The surface area and volume of the individual trihedron is determined as follows:

1) Find the length of each side of the reference plane triangle...

$$a_r = \sqrt{\left(B_x - C_x\right)^2 + \left(B_y - C_y\right)^2}$$

$$b_r = \sqrt{\left(A_x - C_x\right)^2 + \left(A_y - C_y\right)^2}$$

$$c_r = \sqrt{\left(A_x - B_x\right)^2 + \left(A_y - B_y\right)^2}$$

where: a_r = length of Side A
b_r = length of Side B
c_r = length of Side C
A_x = northing coordinate of Point A
A_y = easting coordinate of Point A
B_x = northing coordinate of Point B
B_y = easting coordinate of Point B
C_x = northing coordinate of Point C
C_y = easting coordinate of Point C

2) Calculate the area of the reference plane triangle...

$$s_r = 0.5(a_r + b_r + c_r)$$

$$A_r = \sqrt{s_r(s_r - a_r)(s_r - b_r)(s_r - c_r)}$$

where: A_r = Area of reference plane
s_r = semiperimeter
a_r = length of Side A
b_r = length of Side B
c_r = length of Side C

3) Calculate the height of each flag above the reference plane...

$$h_A = h_{As} - h_r$$
$$h_B = h_{Bs} - h_r$$
$$h_C = h_{Cs} - h_r$$

where: h_A = height of Point A above reference plane
h_B = height of Point B above reference plane
h_C = height of Point C above reference plane
h_{As} = surveyed height of Point A
h_{Bs} = surveyed height of Point B
h_{Cs} = surveyed height of Point C
h_r = elevation of the reference plane

4) Average the height of the three points above the reference plane and calculate the volume of the trihedron...

$$v_p = A_r\left(\frac{h_A + h_B + h_C}{3}\right)$$

where: v_p = volume of the trihedron

5) Now that the volume of the trihedron has been calculated, next determine the surface area that is defined by the easting and northing coordinates of the three survey flags. The plane of the triangle defined by the flags will rarely be parallel to the reference plane, hence the area of the surface plane will be larger than the area of the horizontal reference plane. First calculate the slope length of each side of the triangle defined by the surface plane...

$$a_s = \sqrt{a_r^{\,2} + (h_{Bs} - h_{Cs})^2}$$
$$b_s = \sqrt{b_r^{\,2} + (h_{As} - h_{Cs})^2}$$
$$c_s = \sqrt{c_r^{\,2} + (h_{As} - h_{Bs})^2}$$

where: a_s = slope length of side A (surface plane)
b_s = slope length of side B (surface plane)
c_s = slope length of side C (surface plane)
a_r = horizontal length of Side A (reference plane)
b_r = horizontal length of Side B (reference plane)
c_r = horizontal length of Side C (reference plane)
h_{As} = surveyed height of Point A
h_{Bs} = surveyed height of Point B
h_{Cs} = surveyed height of Point C

7) Finally, calculate the area of the triangle defined by the surface plane...

$$s_s = 0.5(a_s + b_s + c_s)$$
$$A_s = \sqrt{s_s(s_s - a_s)(s_s - b_s)(s_s - c_s)}$$

where: A_s = area of the surface triangle
s_s = semiperimeter of surface triangle

8) The surface area of the complete movement profile is then found by the following...

$$A_p = \sum_{i=1}^{n} A_s$$

where: A_p = surface area of the movement profile
A_s = area of the individual surface triangle (i.e., trihedron)
n = number of trihedra in the profile

9) And the volume of the movement profile above the reference plane is found by...

$$V_P = \sum_{i=1}^{n} v_p$$

where: V_P = volume of the movement profile
v_p = volume of the individual trihedron
n = number of trihedra in the profile

The change in mass of the profile is determined by subtracting the volume of the profile for Epoch 1 from the volume for Epoch 0. A positive result indicates a net gain, while a negative result indicates a net loss. As with the interpolation method, the ablation rate can be calculated. Given the change in volume of the movement profile V, the original surface area A_p, and the elapsed time T in hours between survey epochs, the ablation rate per day A is determined by

$$A = 24\left(\frac{\frac{\Delta V}{A_p}}{T}\right)$$

and the resultant ablation rate is 5.41 cm per day. The final data for Profile IV as derived from the trihedral method are shown in Table 3. The volumes and surface areas for the individual trihedra are presented in Appendix 6.

V (m^3)	Surface Area (m^2)	Ablation Rate (cm/day)
-226,985	838,968	5.41

Table 3: Change in mass balance of Profile IV from July 20, 1993 to July 25, 1994 as calculated via the trihedral method.

Interpolation Method vs. Trihedral Method

As seen in the previous discussion, the interpolation and trihedral methods give slightly different results for the ablation of Profile IV during the survey period. This difference is due to the inherent nature of the two methods. The interpolation method attempts to model the true surface of the glacier taking into account the surface slope, both at the surveyed points and in unsurveyed areas between the flags. The trihedral method builds the surface based exclusively on the surveyed points. It does not attempt to construct the true surface, but rather gives a more generalized, simplistic surface model. As a result, the interpolation method gives a greater net loss than the trihedral method. A summary of the data is shown in Table 4.

	Interpolation Method	Trihedral Method	Difference
Net Loss of Firn (m^3)	257,234	226,985	30,249
Daily ablation (cm)	5.33	5.41	

Table 4: Comparison of net loss and daily ablation as derived from the two methods.

The daily ablation rate is calculated to be 5.33 and 5.41 cm/day, via the interpolation and trihedral methods respectively. Obviously, the problem here is to determine which method gives the most accurate results. This can be decided, in part, by calculating the daily ablation rate as derived directly from the mean flag heights during the two survey epochs. Given a mean flag height of 1,107.872 meters for Epoch 0 and a mean flag height of 1,107.615 meters for Epoch 1, the mean daily ablation is 5.14 cm. While the difference between the interpolation and trihedral methods is only 0.8 mm, the ablation rate derived from the interpolation method is closer to the actual mean as determined by the GPS surveys.

Conceptually, the interpolation method is more accurate because it models the surface morphology of the glacier in the areas where easting, northing, and height coordinates were not directly measured. It does this by examining the surface trends contained in the input data and applying a weighted averaging algorithm to derive convex and concave surfaces. The closer a surveyed data point is to a grid node, the more weight it carries in determining the Z value at a particular grid node. Grid nodes farther away from a data point are given less weight. Additionally, as the grid nodes move closer to each other (i.e., as the cell size decreases) the probability that a grid node will coincide with a data point increases. If a data point and a grid node reside at the same X and Y coordinates, the grid node is given a weight of 1.0 and its Z value will match that of the data point. The degree to which the grid node Z values honor the actual data points can be determined by examining the residuals of the interpolated surfaces. The residuals for Epoch 0 and Epoch 1 are presented in Appendix 7. These residuals represent the vertical deviation between a surveyed data point and the interpolated surface at the same X and Y location. For example, the residual of the interpolated surface for Epoch 0 at Flag 3 is 2.8 cm. This means that the interpolated surface at Flag 3 is 2.8 cm above the actual GPS surveyed height of Flag 3. The standard deviation of all residuals for both epochs is 4.2 cm. The accuracy of the GPS surveyed heights is ±5 cm (Lang, 1994). Thus the residuals, and hence the accuracy of the interpolated surface, is within the height tolerance obtainable by the GPS equipment. This indicates that the accuracy of the interpolated surface is indeed within acceptable limits.

Determining the mass balance via the interpolation method is more complicated than it is with the trihedral method. A computer and the appropriate software is necessary, and some experimentation is required to determine the most appropriate interpolation parameters to use. This experimentation requires a knowledge of the various interpolation methods, the way in which each method handles the weighting of grid nodes, and how the search parameters affect the interpolation. One of the greatest benefits of the interpolation method is the ability to create topographic and surface maps, and to view the maps from different perspectives. This allows the researcher to more fully understand the surface morphology of the surveyed profile, and to obtain a greater understanding of the temporal and spatial mass balance changes across the profile.

The trihedral method is easier to perform than the interpolation method because it does not require the use of specialized computer hardware and software. The mass balance can be determined using a basic hand calculator, which means that the mass balance can be calculated in the field. The major drawback to this method is that the derived surface is composed of a series of planar triangles, the points of which are defined by the survey flags. Thus the spacing of the flags determines the resolution, and hence the accuracy, of the derived surface. Ideally, the flags should be spaced 5-10 meters apart, however this is not practical for a 3-4 kilometer profile. For a profile of such length, logistics dictate a flag spacing of 150-200 meters. The derived surface is therefore constrained by the flag spacing and is not able to accurately depict convex and concave variations in the surface morphology between the flags. This is illustrated in Figure 5.

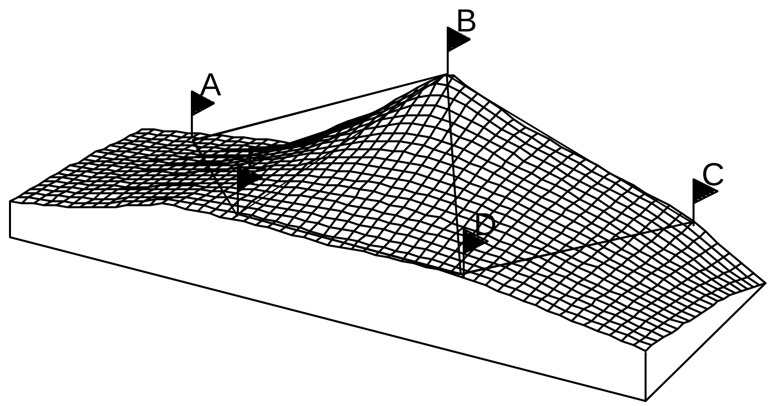

Figure 5: Flag spacing determines the accuracy of the trihedral derived surface. Large spacings cannot detect convex or concave surfaces between the flags, as shown by trihedra ABE and BCD. Interpolation, as represented by the grid, can construct a surface with convex and concave surfaces that more closely approximates the true surface.

As shown in Figure 5, the trihedral method constructs a more generalized surface than does the interpolation method. Because the flag spacings must be relatively far apart, the trihedron boundaries do not exactly follow the true surface contours. Line AB, in particular, is above the interpolated surface, while line BC is above the surface in one location and below in another. This graphically illustrates the low resolution, planar nature of the trihedral method. Conversely, the interpolation method is able to produce a high resolution surface model, and in fact the accuracy of the surface interpolation increases as the resolution increases (i.e., as the cell size decreases).

Considering the advantages and disadvantages of both methods, the interpolation method provides a better overall solution for determining the mass balance of a GPS surveyed profile. It constructs a surface that is a closer approximation of the true surface than does the trihedral method. Additionally, the trihedral method will always show a smaller surface area and a greater ablation rate than the interpolation method. The trihedral method is good for rough mass balance determinations, however in order to calculate mass balance that is closer to the true mass balance state, the interpolation method is needed.

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APPENDICES

Appendix 1
GPS Derived Easting, Northing, and Height Coordinates for Taku Profile IV

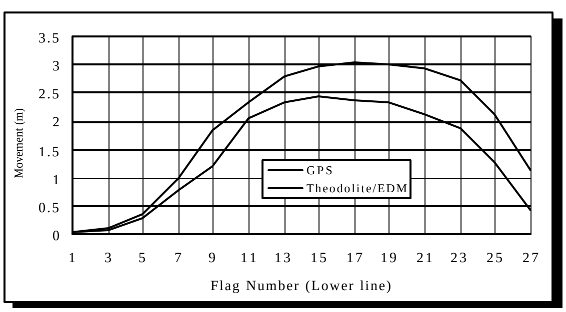
Flag	Epoch 0: July 20, 1993			Epoch 1: July 25, 1993		
	Easting	Northing	Height	Easting	Northing	Height
1	487,759.138	6,503,058.680	1,100.274	487,759.200	6,503,058.661	1,100.112
2	487,541.994	6,503,210.228	1,107.433	487,542.010	6,503,210.166	1,107.183
3	487,615.859	6,502,929.145	1,103.261	487,615.979	6,502,929.080	1,103.092
4	487,394.822	6,503,060.300	1,106.381	487,394.916	6,503,060.215	1,106.142
5	487,468.877	6,502,796.231	1,103.291	487,469.160	6,502,795.977	1,103.110
6	487,233.799	6,502,896.364	1,103.871	487,234.200	6,502,895.997	1,103.744
7	487,281.323	6,502,626.819	1,101.111	487,282.078	6,502,626.155	1,100.916
8	487,094.073	6,502,753.472	1,103.043	487,094.911	6,502,752.706	1,102.805
9	487,103.563	6,502,465.837	1,101.705	487,104.940	6,502,464.587	1,101.461
10	486,950.825	6,502,607.812	1,102.885	486,952.221	6,502,606.589	1,102.654
11	486,969.926	6,502,344.990	1,102.126	486,971.669	6,502,343.424	1,101.917
12	486,769.663	6,502,422.365	1,102.167	486,771.474	6,502,420.734	1,101.935
13	486,731.221	6,502,128.404	1,102.044	486,733.307	6,502,126.569	1,101.826
14	486,498.331	6,502,202.455	1,103.932	486,500.478	6,502,200.626	1,103.643
15	486,499.275	6,501,918.988	1,098.486	486,501.526	6,501,917.076	1,098.205
16	486,237.357	6,501,975.229	1,103.094	486,239.620	6,501,973.374	1,102.731
17	486,207.970	6,501,654.995	1,102.569	486,210.344	6,501,653.112	1,102.158
18	485,906.422	6,501,674.180	1,108.954	485,908.849	6,501,672.337	1,108.591
19	485,930.605	6,501,403.099	1,108.859	485,932.986	6,501,401.266	1,108.565
20	485,654.529	6,501,446.238	1,114.883	485,656.900	6,501,444.443	1,114.520
21	485,651.222	6,501,149.361	1,115.235	485,653.565	6,501,147.620	1,114.941
22	485,406.094	6,501,224.219	1,119.186	485,408.380	6,501,222.520	1,118.856
23	485,412.345	6,500,932.607	1,117.489	485,414.497	6,500,930.948	1,117.195
24	485,136.151	6,500,996.222	1,119.836	485,138.212	6,500,994.713	1,119.594
25	485,124.967	6,500,671.377	1,119.252	485,126.708	6,500,670.139	1,118.994
26	484,873.851	6,500,781.423	1,121.358	484,875.388	6,500,780.362	1,121.100
27	484,844.499	6,500,417.149	1,119.813	484,845.470	6,500,416.570	1,119.610

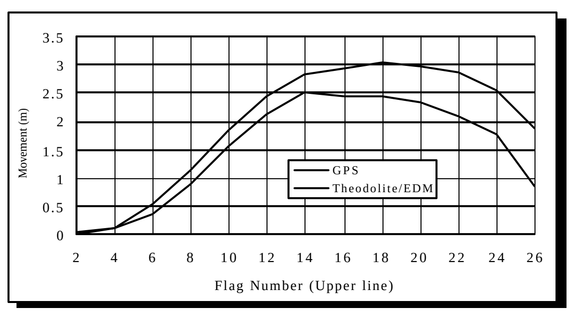
Appendix 2
Theodolite/EDM Derived Easting, Northing, and Height Coordinates for Taku Profile IV

Flag	Epoch 0: July 20, 1993			Epoch 1: July 25, 1993		
	Easting	Northing	Height	Easting	Northing	Height
1	99,765.602	99,741.923	1,116.332	99,765.642	99,741.892	1,116.113
2	99,535.371	99,872.709	1,123.470	99,535.396	99,872.699	1,123.182
3	99,634.935	99,599.709	1,119.350	99,635.015	99,599.681	1,119.151
4	99,402.651	99,709.833	1,122.439	99,402.742	99,709.783	1,122.226
5	99,500.836	99,453.739	1,119.378	99,501.108	99,453.604	1,119.166
6	99,257.506	99,531.643	1,119.983	99,257.800	99,531.403	1,119.782
7	99,329.817	99,267.610	1,117.283	99,330.490	99,267.189	1,117.034
8	99,131.545	99,376.339	1,119.112	99,132.301	99,375.848	1,118.873
9	99,167.705	99,090.856	1,117.875	99,168.773	99,090.288	1,117.568
10	99,002.385	99,218.047	1,118.932	99,003.746	99,217.250	1,118.750
11	99,045.772	98,958.147	1,118.346	99,047.438	98,956.947	1,118.114
12	98,839.123	99,016.555	1,118.285	98,840.858	99,015.354	1,118.031
13	98,828.101	98,720.291	1,118.226	98,830.050	98,719.017	1,118.026
14	98,589.302	98,772.512	1,119.315	98,591.349	98,771.066	1,119.766
15	98,616.536	98,490.333	1,114.649	98,618.559	98,488.990	1,114.391
16	98,350.377	98,522.069	1,119.214	98,352.456	98,520.780	1,118.971
17	98,350.836	98,200.486	1,118.811	98,352.845	98,199.257	1,118.496
18	98,048.682	98,191.635	1,125.122	98,050.817	98,190.442	1,124.781
19	98,097.898	97,923.900	1,125.173	98,099.952	97,922.784	1,124.781
20	97,818.917	97,941.276	1,131.184	97,821.016	97,940.284	1,130.863
21	97,843.144	97,645.440	1,131.721	97,845.039	97,644.517	1,131.278
22	97,591.973	97,697.282	1,135.517	97,593.852	97,696.371	1,135.265
23	97,625.237	97,407.462	1,133.930	97,626.947	97,406.647	1,133.595
24	97,344.209	97,445.232	1,136.263	97,345.826	97,444.538	1,136.004
25	97,363.188	97,120.746	1,135.664	97,364.400	97,120.290	1,135.571
26	97,102.952	97,207.030	1,137.724	97,103.792	97,206.849	1,137.716
27	97,107.254	96,841.719	1,136.331	97,107.633	96,841.904	1,136.234

Appendix 3
Comparison of GPS and Theodolite/EDM Derived Movement Vectors for Taku Profile IV

Flag	GPS Method		Theodolite/EDM Method	
	Movement (m)	Bearing (gons)	Movement (m)	Bearing (gons)
1	0.065	118.9308	0.051	141.9729
2	0.064	183.9217	0.027	124.2237
3	0.136	131.6032	0.085	121.4333
4	0.127	146.8017	0.104	131.9851
5	0.380	146.5653	0.304	129.3291
6	0.544	147.1834	0.380	143.5840
7	1.005	145.9229	0.794	135.5870
8	1.135	147.1442	0.901	136.6694
9	1.860	146.9246	1.210	131.1173
10	1.856	145.8008	1.577	133.7257
11	2.343	146.5978	2.053	139.7385
12	2.437	146.6737	2.110	138.5463
13	2.778	145.9302	2.328	136.8570
14	2.820	144.9190	2.506	139.1526
15	2.953	144.8272	2.428	137.3097
16	2.926	143.7130	2.446	135.3325
17	3.030	142.6895	2.355	134.9511
18	3.047	141.3467	2.446	132.4396
19	3.005	141.7673	2.338	131.6850
20	2.974	141.2533	2.322	128.1063
21	2.919	140.6829	2.108	128.8548
22	2.848	140.6893	2.088	128.7395
23	2.717	141.8099	1.894	128.3142
24	2.554	140.2337	1.760	125.8094
25	2.136	139.3511	1.295	122.9091
26	1.868	138.4639	0.859	113.5110
27	1.131	134.2303	0.422	71.0907



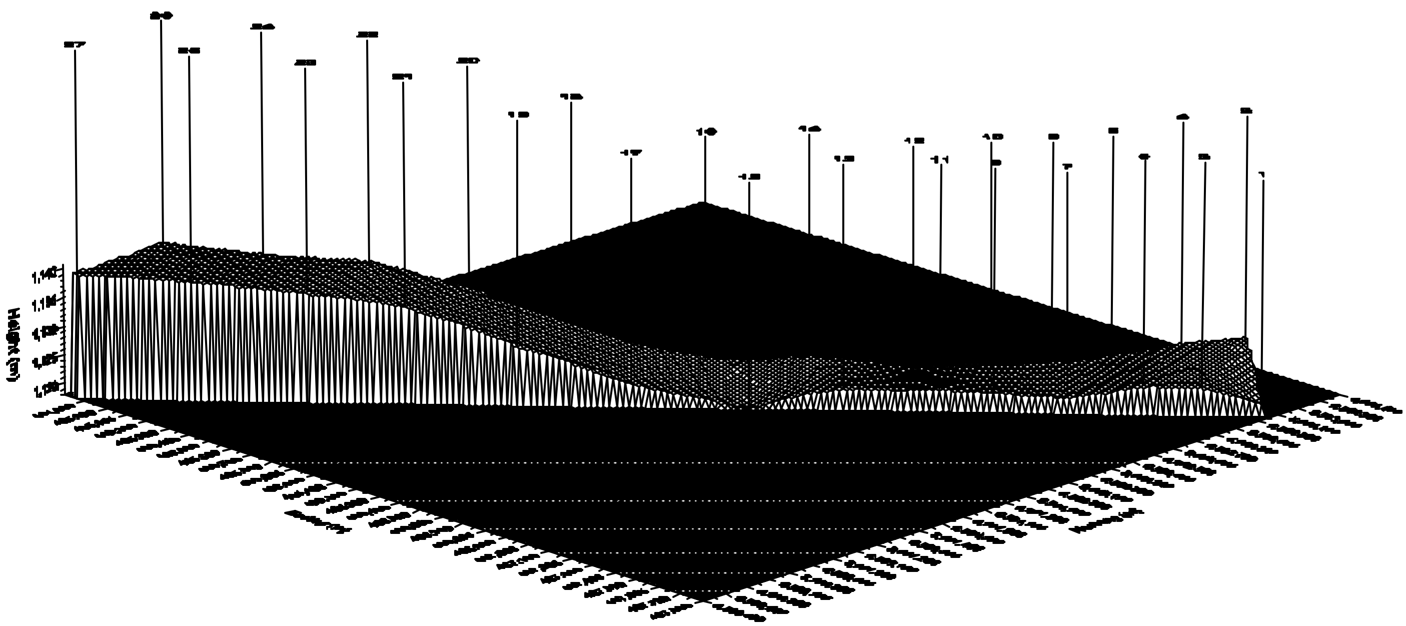


Comparison of GPS and theodolite/EDM derived movement vectors for Profile IV. The GPS movement data is more accurate as evidenced by the smoother and more consistent movement vectors. Additionally, movement detected by GPS is greater than that obtained by theodolite/EDM methods.

Appendix 4
Blanking File Coordinates Used in the Generation of Final Blanked Grids
Taku Profile IV (7-20-93 to 7-25-93)

Easting	Northing	Point
487,776	6,503,052	1
487,632	6,502,908	3
487,488	6,502,776	5
487,296	6,502,608	7
487,116	6,502,452	9
486,984	6,502,332	11
486,744	6,502,116	13
486,516	6,501,900	15
486,228	6,501,636	17
485,952	6,501,384	19
485,664	6,501,132	21
485,436	6,500,916	23
485,148	6,500,652	25
484,848	6,500,400	27
484,824	6,500,400	
484,860	6,500,796	26
485,124	6,501,012	24
485,388	6,501,240	22
485,640	6,501,468	20
485,892	6,501,696	18
486,216	6,501,996	16
486,480	6,502,224	14
486,756	6,502,440	12
486,936	6,502,620	10
487,080	6,502,776	8
487,212	6,502,920	6
487,380	6,503,076	4
487,536	6,503,232	2
487,776	6,503,064	
487,776	6,503,052	1

Appendix 5
Interpolated Surface of Taku Profile IV



Taku Glacier Profile IV

Surface of Profile IV as derived from the interpolation method. This shows the surface as surveyed on July 20, 1993. The surface as surveyed five days later is similar, however it is not shown because the elevation difference is too small be be depicted at this scale. Vertical scale is exaggerated 20x.

Appendix 6
Profile IV Net Loss (July 20, 1993 to July 25, 1993)
Calculated via the Trihedral Method

	Epoch 0: July 20, 1993		Epoch 1: July 25, 1993	
Trihedron	Surface Area (m^2)	Volume (m^3)	Surface Area (m^2)	Volume (m^3)
1 2 3	24,930.961	215,713.553	24,931.249	210,891.480
2 3 4	26,223.789	280,346.079	26,221.582	274,573.006
3 4 5	24,330.118	226,519.572	24,340.497	221,838.491
4 5 6	27,332.979	260,033.690	27,331.847	255,041.113
5 6 7	29,304.397	227,320.497	29,308.428	222,437.545
6 7 8	22,227.803	170,589.324	22,224.467	166,415.137
7 8 9	26,329.823	183,064.552	26,329.516	177,121.231
8 9 10	21,293.076	160,639.422	21,288.910	155,548.366
9 10 11	18,715.853	135,475.419	18,714.666	131,199.862
10 11 12	25,577.914	189,088.084	25,590.515	183,449.020
11 12 13	30,921.984	219,927.434	30,920.529	213,124.882
12 13 14	35,654.704	275,043.932	35,659.341	266,296.469
13 14 15	32,979.509	213,909.195	32,976.632	205,230.532
14 15 16	37,106.191	253,637.780	37,112.227	242,141.475
15 16 17	42,770.232	272,961.984	42,769.464	257,920.077
16 17 18	48,575.818	479,448.446	48,571.255	460,997.324
17 18 19	40,649.234	479,308.057	40,640.674	464,740.337
18 19 20	36,906.900	586,626.838	36,907.250	574,089.477
19 20 21	41,061.559	738,614.749	41,056.074	725,507.654
20 21 22	36,515.153	782,584.188	36,518.074	770,634.782
21 22 23	35,511.403	791,928.125	35,519.617	781,245.385
22 23 24	40,073.593	955,194.206	40,085.174	943,898.672
23 24 25	45,217.406	1,078,805.086	45,205.859	1,066,563.908
24 25 26	41,403.513	1,041,211.780	41,421.214	1,031,191.611
25 26 27	47,354.025	1,190,490.459	47,364.390	1,179,400.072
Total	838,967.937	11,408,482.451	839,009.451	11,181,497.908

	Surface Area (m^2)	Volume (m^3)
July 20, 1993	838,967.937	11,408,482.451
July 25, 1993	839,009.451	11,181,497.908
Net Gain <Loss>	41.514	<226984.543>

Appendix 7
Residuals of Interpolated Surfaces for Taku Profile IV

Flag	July 20, 1993	July 25, 1993
1	-0.124	-0.123
2	0.084	0.083
3	0.028	0.028
4	0.023	0.023
5	0.025	0.025
6	-0.011	-0.008
7	-0.040	-0.037
8	0.001	-0.002
9	0.001	0.000
10	0.003	0.005
11	0.012	0.004
12	-0.036	-0.041
13	0.007	0.014
14	0.052	0.037
15	-0.111	-0.097
16	0.000	0.000
17	-0.038	-0.043
18	0.012	0.014
19	-0.024	-0.027
20	0.023	0.024
21	0.003	0.003
22	0.051	0.050
23	-0.008	-0.010
24	-0.003	-0.003
25	-0.001	-0.001
26	0.015	0.021
27	-0.004	-0.003

The residuals indicate the degree to which the interpolated surface honors the original surveyed data points. It is the vertical deviation, in meters, of the surface either above or below the actual surveyed flag height. A positive residual indicates the surface is above the surveyed flag height, while a negative residual places the surface below the surveyed flag height. As the residuals approach zero, the accuracy of the interpolated surface increases.